Universal Software to Modelling, Processing and Adjusting of Heterogeneous Multiepoch and Permanent Geodetic Observations

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SUMMARY

Scientific and technical problems in geodesy require processing of various types of geodetic observations. The article aims to introduce software, which does not require the a-priori information about type of geodetic observations. Definition of mathematical model for geodetic network is realized only through observation equation, which relates the observations with adjusted unknown parameters. The quality of numerical stability of the solved geodetic network plays very important role too. In addition, estimated coordinates and other related parameters can be quickly and easily transformed to various coordinate systems for better interpretation of results obtained.

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1. INTRODUCTION

Most of geodetic software, dealing with geodetic networks adjustment allows processing of geodetic networks of the specific type, e.g. if the geodetic software is built for an adjustment of the geodetic networks observed by terrestrial techniques, it does not permit processing of permanent satellite networks.

Our aim is to build a universal software package, which will permit the processing of various geodetic observations without any a-priori definition of type of the geodetic network. The only requirement is the knowledge of the symbolic form of the observation equations. This solution will provide to user a tool, which permits to process every kind of geodetic network. He has only to compile the observation equations, which means that he has to define the functional relations between the observables and the unknown parameters.

We developed a software solution named Sonet (Solution Networks). It can be used for processing and adjustment of geodetic networks based on satellite technologies observations, either epoch or permanent, as well as for networks built only by classical terrestrial methods or by combination of satellite and terrestrial methods. Universality of the programme is emphasised by the fact that it also allows solving stochastic and deterministic transformations too.

2. DATA STRUCTURE OF THE PROGRAMME

The basic feature of the Sonet programme is that it enables processing of multistage heterogeneous geodetic observations without specific information about the type of the geodetic network.

The programme Sonet performs all operations over its data structure and therefore it is necessary to explain its functions in main features. This is crucial, because user of the programme forms the structure of the geodetic network by defining an observation equation or rather the observation equations in the symbolic form. All information needed for compiling the geodetic network along with the observations themselves is assigned into an input file, which represents an XML batch (Harold, 2002).

The data structure is divided into *the data structure of the geodetic observations* and *the data structure of the topology of the geodetic network*. The principle and functioning of the data structure of the observations is described in Fig. 1.

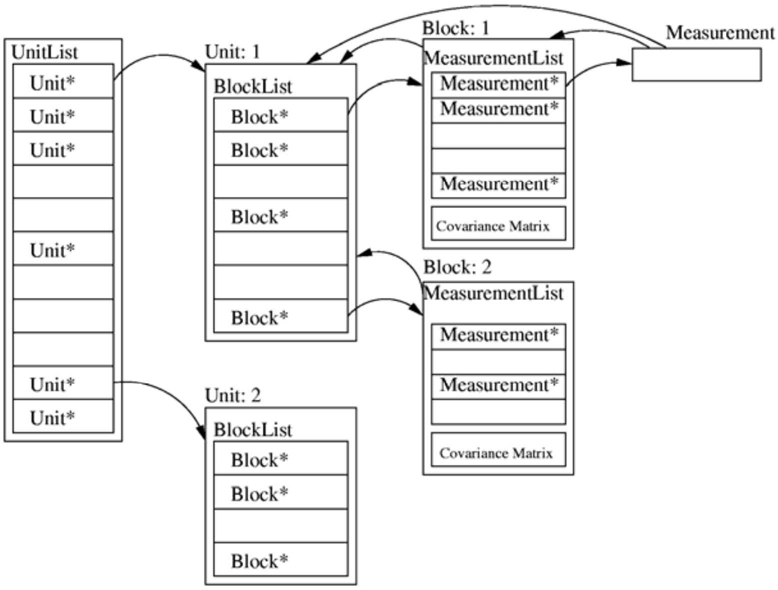

Fig. 1: The data structure of the geodetic observations.

The list *Unitlist* (the first input hierarchic node) is the input node of the data structure of the programme Sonet and it contains the data structure of the separate epochs *Unit*. Of course, one or more epochs can exist; the extent is dependent only on the computer memory. Every epoch contains a block list *BlockList* (the second hierarchical node) where every block contains a list of observations *MeasurementList* (the third hierarchical node). In general we can denote the blocks as a module, which encapsulates optional geodetic observations. They have their own covariance matrix *CovarianceMatrix* (diagonal, dense, ...). The meaning of the blocks is in combination of various types of geodetic observations e.g. satellite and terrestrial within the scope of one epoch.

The data structure of the topology of the geodetic network provides three basic functions of the programme. The hierarchic arrangement is similar to the case of the data structure of the geodetic observations and is divided into three hierarchic stages again. The interface between the data structures of the geodetic observations and the date structure of the topology of the geodetic networks is managed by a part of the programme named *SnJoiner*.

The first function of the topology is to define a selection of points and corresponding observations for processing. The importance of the selection of points lies in simple manipulation with observations, which we want to incorporate into the adjustment. The selection of points or rather observations can be made for each of the hierarchic nodes. In the selection of points, blocks or units the *regular expressions* can be fully taken with advantage (Friedl, 1997). This substantially simplifies work especially in the geodetic network with considerable number of points, epochs or blocks.

The second function concerns the definition or rather the selection of unknowns for the input node, individual epochs (the second hierarchic node) and individual blocks (the third hierarchic node). The introduction of this function is to add new estimated parameters similarly the new observations are included. The most essential property in defining the unknowns is variability of their selection. As the unknowns related to single points can be selected, also in reverse, points to which the unknowns are related can be selected too. It means that for different points we can define different unknowns.

We do not always want to form a new unknown for each point. For example, in transformation, we do not want to form translation parameters for every point or we want to set the estimated velocity for the determinate group of points. For this reason the unknowns can be classified to groups. The groups are divided into *anonymous* and *named* ones. The anonymous groups indicate a fact that for every point, which is enclosed in it, an independent variable is created. The named group indicates that for the points enclosed in the group only one variable is formed. The points enclosed in both types of the groups can be mutually combined.

The selection of unknowns is realized only after reading in all observations into the programme. It realises when the units, blocks and points, which observations are connected with, are not represented by regular expressions but its real names. This is why the use of regular expressions has relation to the selected points and not to the original ones.

Finally we can set the observation equations into the programme (the third function). The observation equations represent the most progressive part of the programme Sonet. The part referring to the observation equations has a close relation with the part where the unknowns are defined. The observation equations in geodetic networks connect directly observed values with unknowns, which are an object of the adjustment. The procedure in „normal" processing is that if the observation equation does not have a linear form it is necessary to perform a linearization. The development into Taylor series is used where higher derivations are disregarded as we expect that we work with parameters close to their approximate values (Kubáčková, 1990).

The observation equations of course can be defined for the optional input node it means for the root input node of the units and for input node of the blocks. The observation equations are inherited between every single node by system from above down. The term „the inheritance from above down" means if we defined the observation equations in determined node it concerns for the subordinate node too. The advantage of the solving is that the equations have been written in determined node we have not to write in all hierarchic lower nodes. If the observation equation is defined on the hierarchic lower position just the equation will be used for this node and lower node and not the equation defined by level higher. On the Fig. 2 we have defined the observation equation in the node 1, 2 and 3.

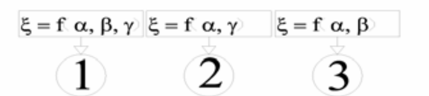

Fig. 2: Symbolic description of the observation equation in hierarchic nodes.

The node 1 is hierarchic the highest node and the node 3 is hierarchic lowest node but with the highest priority. In every node is defined the observation equation for measured value ξ and unknowns α, β and γ.

If the observation equation would be defined only in the node 1 the programme would form the same observation equations for the nodes 2 and 3 too. Because the observation equation are defined in the every node in this case the programme understands the situation that for the data in the node 1 will use the equation defined in the node. For the data related to the node 2 will use the observation equation defined in this node. The programme is performing the same in the case of the node 3.

2. MODEL EXAMPLES

The versatility of the programme Sonet we can demonstrate in its practical using. The model examples will be shortened on minimum to stress the nature of the problem.

2.1 Levelling Network

Probably the simplest example, which is possible to form, is processing of one-stage free levelling network. In the shown example the input data are not illustrated, nor the estimated heights of the points, because the example is focused to forming the geodetic networks in the programme Sonet.

As for whole levelling network only one observation equation is valid it is advantageous to define it on the highest hierarchic stage. The observation equation connects unknown difference in height with the estimated heights of the points. The unknown variables we denote by symbol H. For this unknown we form one unnamed group because we ask to form one variable for each point. We select the points in this group by regular expression “all accessible points“. Finally we form one epoch.

```
<compute>
  <unknowns>
    <unknown name="H"> <!-- we need create unknown H -->
      <group>
        <point name=".*"/> <!-- for each point will be created one unknown named H -->
      </group>
    </unknown>
  </unknowns>
  <equations>
    <equation formula="h{i,j} = H{j} – H{i}"/> <!-- observation equation -->
  </equation>

  <measurements>
    <unit id="1" state="pass">
      <measurements>
        <point name="[A-Z].*" state="select"/> <!—sel. only points begin. with big letter -->
      </measurements>
    </unit>
</compute>
</options>
```

2.2 Stochastic Transformation of the Coordinates

We often meet in multistage networks the situation when individual stages are related to different reference frames. The mathematic model will contain apart from the estimate of the coordinates additional parameters, which define the relation between the reference frames in the first and the second stages. We mark such model like an *effective transformation*. The mathematic model has the form (Hefty, 2003):

$$\begin{pmatrix} \mathbf{x}^{(1)} \\ \mathbf{x}_I^{(2)} \\ \mathbf{x}_N^{(2)} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_I & \mathbf{0} & \mathbf{0} \\ \mathbf{I}_I & \mathbf{0} & \mathbf{T}_I \\ \mathbf{0} & \mathbf{I}_N & \mathbf{T}_N \end{pmatrix} \begin{pmatrix} \mathbf{y}_I \\ \mathbf{y}_N \\ \mathbf{\Theta} \end{pmatrix} \tag{2.1}$$

Where $\mathbf{x}_I^{(i)}$ is the vector of the realization of the identical points in *i*-th stage, $\mathbf{x}_N^{(i)}$ is the vector of the non-identical points in *i*-th stage. $\mathbf{I}_I$ and $\mathbf{I}_N$ are identity matrices with dimensions corresponding the numbers of the elements vector identical and non-identical points, $\mathbf{y}_I$ and $\mathbf{y}_N$ are the vectors of the coordinates of identical and non-identical points which we have achieved by combination mentioned of two stages of the observation network. $\mathbf{T}_I$ and $\mathbf{T}_N$ are matrices defining the relation between the coordinates of identical and non-identical points. $\mathbf{\Theta}$ is matrix which contains the transformation parameters. The concrete form of the matrix **T** and the vector depends on the selected transformation model. In our example we will apply the transformation marked as Burša`s and Wolf`s model (we will not include the scale factor). The matrix **T** has a form:

$$T = \begin{pmatrix} \mathbf{T}_1 \\ \mathbf{T}_2 \\ \vdots \\ \mathbf{T}_p \end{pmatrix}, j = 1,2,\cdots,p, \mathbf{T}_j = \begin{pmatrix} 1 & 0 & 0 & 0 & -Z_j & Y_j \\ 0 & 1 & 0 & Z_j & 0 & -X_j \\ 0 & 0 & 1 & -Y_j & X_j & 0 \end{pmatrix} \tag{2.2}$$

where p is the number of the identical (or non identical) points and (X_j, Y_j, Z_j) are the geocentric coordinates *i*-th point. The vector of the parameter has a form:

$$\mathbf{\Theta} = (\Delta X \quad \Delta Y \quad \Delta Z \quad \omega \quad \psi \quad \varepsilon)^T \tag{2.3}$$

It defines the position of the origin and rotation around three coordinate axes of the coordinate frame of the second stage with regard to the position of the coordinate frame of the first stage. The mathematic model is now defined and we need further to form the observation equations.

The observation equations will be 3 in the first stage (for the coordinate X, Y, Z) and 3 in the second stage (for coordinate X, Y, Z). The observation equations for the first stage will have the form:

$$\begin{aligned} X^{(1)} &= X \\ Y^{(1)} &= Y \\ Z^{(1)} &= Z \end{aligned} \tag{2.4}$$

where *X*, *Y*, *Z* are the estimated coordinates and $X^{(1)}, Y^{(1)}, Z^{(1)}$ are the coordinates of the vector realization of the first stage. The observation equations for the second stage will have a form:

$$
\begin{aligned}
X^{(2)} &= \Delta X + X + \omega Y - \psi Z \\
Y^{(2)} &= \Delta Y - \omega X + Y + \varepsilon Z \\
Z^{(2)} &= \Delta Z + \psi X - \varepsilon Y - Z
\end{aligned}
\tag{1.5}
$$

where *X*, *Y*, *Z* are the estimated coordinates, $X^{(2)}, Y^{(2)}, Z^{(2)}$ are coordinates of the vector realization of the second stage, $\Delta X, \Delta Y, \Delta Z$ are three translations and $\varepsilon, \psi, \omega$ are rotation angles .

When forming the input file for Sonet we have to be aware that we estimate the coordinates for each point individually but we determine the translation parameters and rotation angles only once for all points. With advantage we use the unification into the group. In the case we will expect that the identical points are only in the first stage and in the second stage identical and non-identical points. If not, we have to write only those points in the configuration file, which are identical. In next part of the configuration file is not shown the part in which the input dates are defined. In an extract we suppose that the identical points start by a small letter and the non-identical points start by a capital letter.

```
<compute>
  <unknowns>
    <unknown name="X:Y:Z">
      <group>
        <point name=".*"/>
      </group>
    </unknown>
  </unknowns>
  <measurements>
    <unit id="first" state="pass">
      <equations>
        <equation formula="coordinateX{i} = X{i}"/>
        <equation formula="coordinateY{i} = Y{i}"/>
        <equation formula="coordinateZ{i} = Z{i}"/>
      </equations>
      <measurements>
        <point name="[a-z].*" state="select"/>
      </measurements>
    </unit>

    <unit id="second" state="pass">
      <equations>
        <equation formula="coordinateX{i} = TX{} + X{i} + omega{}*Y{i} - psi{}*Z{i}"/>
        <equation formula="coordinateY{i} = TY{} - omega{}*X{i} + Y{i} + epsilon{}*Z{i}"/>
        <equation formula="coordinateZ{i} = TZ{} - psi{}*X{i} - epsilon{}*Y{i} + Z{i}"/>
      </equations>
      <unknowns>
        <unknown name="TX:TY:TZ:omega:psi:epsilon">
          <group name="my own group"> <!-- create only one unknown for another points -->
            <point name="[A-Z].*"/> <!-- select only nonidentical point -->
          </group>
        </unknown>
      </unknowns>
      <measurements>
        <point name=".*" state="select"/>
      </measurements>
    </unit>
  </measurements>
</compute>
```

The result will be the estimate of the coordinates of the identical and non-identical points, the translation parameters and the rotation angles.

Probably the most elegant would be an example of the common processing of the geodetic networks measured by the satellite technologies and the geodetic networks measured by terrestrial methods. As this problem is more complex, such example is not relevant for demonstration.

3. DETERMINICTIC TRANSFORMATION OF THE COORDINATES

A lot of the tasks processed in Sonet are solved in the 3-dimensional geocentric coordinate system. All the information that form the input of the programme is necessary to set in the geocentric coordinates. It depends on the type of the task solved because Sonet is generally independent on the coordinate system.

The philosophy of using the Cartesian coordinate system has an importance in the combination of the satellite and terrestrial observations. Then all information obtained is related to geocentric coordinate system, which has deficiency concerning the interpretation of obtained results. For purpose of more illustrative interpretation a series of deterministic transformations has to be performed. The transformations of Cartesian coordinates realised by Sonet will be describe in the next text.

The first stage is the transformation of the coordinates (X, Y, Z) into the elipsoidal coordinates (B, L, H). The relation between Cartesian coordinates and geodetic (ellipsoidal) coordinates arises from geometry of the rotation ellipsoid. The name of the applied ellipsoid is presented in the configuration file. Sonet contains the parameters of most relevant ellipsoids. In case of selection of a non-standard ellipsoid it is possible to write its parameters into the configuration file. The selection of the method of the transformation is possible in the configuration file. Apart from the coordinates also their covariance matrix is transformed.

The second stage is the transformation of the Cartesian coordinates (X, Y, Z) or rather of the differences (ΔX, ΔY, ΔZ) into the local topocentric system (n, e, v) where are used the parameters obtained from the transformation of the first stage. The important step in the stage is the selection of the origin or rather the origins of the local coordinate system. The transformation of the covariance matrix is performed too.

4. SUBSIDIARY INFORMATION FILES

Sonet uses auxiliary information files. There are the text files in the XML format. These files are two in the current version of the programme Sonet. The first file contains the constants of the ellipsoids. Of course the constants of the other ellipsoids can be implemented. We refer to the ellipsoid in the input file of the programme Sonet according to its name. The second file contains the tectonic plate motion models (DeMets, 1994). In the file are implemented the models NUVEL-1, NNR-NUVEL-1, NUVEL-1A, NNR-NUVEL-1A a APKIM2000.0. The file is in the text form in the XML; the adding another model is trouble-free. The model and the plate in the input Sonet file is refer by its name. In the names we distinguish the small and capital letters.

5. NUMERIC ASPECT OF THE PROGRAMME

The programme processes the big amount of data, first of all in the situations if we take into consideration the networks with the huge number of the points, if we estimate a lot of unknown parameters or we process the multistage geodetic networks. A lot of epochs are reflected mainly in the processing of the permanent geodetic networks where hundreds epochs are processed. We mostly consider dense covariance matrices. The programme Sonet uses the library uBLAS and the library CLAPACK (Anderson, 1999). The user has a possibility to select in the input file of the programme Sonet the algorithm with which the calculation can be realised. There are the algorithms in the current version a) Gram-Schmidth's ortogonalization, b) Gauss-Jordan's algorithm with the full pivoting, c) modified Gram-Schmidth's algorithm and d) Singular value decomposition (Golub, 1989).

6. CONCLUSION

We are able to process in the system Sonet geodetic networks and geodetic tasks in which we can compose the function relations between the directly observed values and the estimated unknowns. The programme can be applied in the processing, analyse and the adjustment of geodetic networks. The independence on the type of processed geodetic networks enables to estimate the optional parameters and their combinations. In this way is possible to solve very simply the combinations of satellite and terrestrial geodetic networks with the assessment useless parameters, to solve the stochastic transformations, to estimate a velocity of the points or e.g. to process the permanent networks. Of course, the application can be found in the processing only terrestrial networks too, but the current geodetic software solves this satisfactory. In the combination of geodetic networks measured by the satellite technologies and geodetic networks measured by the classical terrestric methods, in processing of the permanent networks etc., the obtained results are in the geocentric Cartesian coordinate system. The obtained results have less the interpretative ability. For better interpretation of the results the deterministic transformations are part of the programme.

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