The impact analysis on the positioning of moving station resulted from the datum inconformity in RTK

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KEY WORDS: RTK; Difference positioning; Precision ephemeris; Difference correction; Datum inconsistency

ABSTRACT:

In the real-time precise positioning system depending on the RTK technology, it is necessary to get the precise geocentric coordinate of benchmark stations in advance by precision post-processing method. For the convenience to users, the coordinate system of base station is China Geodetic Coordinate System 2000 (CGCS2000). In the process of Network RTK, it is needed to adopt the precise ephemeris provided by IGS or satellite broadcast ephemeris. The benchmark is ITRF or WGS84 respectively, which lead to the inconsistency of datum between fiducial station coordinate system and the satellite orbit. So there is a certain influence to the positioning accuracy of rovers. Based on the principle analysis of difference positioning, this paper deduced the relationship on positioning accuracy among the satellite's orbit, the coordinate of fiducial station and the moving station utilizing the analytical method. Addition, the relationship between the positioning accuracy of rovers and the distance to base station was analyzed, which provide the beneficial discussion to promote the use of RTK technology.

1. INTRODUCTION

Difference positioning is a very useful method for high precise positioning. This is widely used in RTK. It requires provide precise benchmarks coordinates for GPS base station. Recently, many countries are actively promoting the respective national geocentric coordinate reference framework system. In our country, State Bureau of Surveying and Mapping issued "Start-up on CGCS2000 implementation scheme" in which explicitly pointed out that the established urban GPS control network geocentric coordinate in the provinces and cities should convert to ITRF97 framework on J2000.0. The converted coordinate passes for the results under the CGCS200.

At present, the continuous operation reference system has been built in the provinces and cities in China are based on GPS satellite system. The precise base station coordinate given under the system is CGCS2000 coordinate. However, the ephemeris used is broadcast ephemeris or precision ephemeris when positioning. Both of the ephemeris is based on coordinate reference frame of ITRF or WGS-84, which results in the inconsistency on coordinate frame. Whether the datum inconstancy leads to significant influence to positioning, this paper gave thorough discussion from the RTK principle and difference theory. Then it provided a useful theoretical guidance for the future high precision positioning.

2. EXPRESSION ON PRINCIPLE

2.1 RTK Principle

When computing the coordinate of a station using the difference method, if the coordinate of another station is precisely known, a more precise coordinate can be got. So for the RTK, the benchmark station coordinate is precisely known and the coordinate general is determined by the long time GPS static relative positioning method. The basic principle is that the benchmark obtains the continuous observation under a fixed sampling epoch and then transmitted the real-time observation data to data processing center through the data communication network. At the same time, rover station transmits the observation to data processing center. According to the base station observation and the approximate coordinate provided by rover station, data processing center computes the system error on rover station. Then the system error is spread to user for revising the observation of rover station so as to reaches an accurate result.

2.2 Comparison on CGCS2000 with other Coordinate system

The definition of CGCS2000 includes original point, axis of coordinate, scale and directed temporal evolution definition. The definition is as the same as WGS-84.

CGCS2000 is realized by the coordinate and velocity of national GPS geodetic network. The geodetic network is composed of the combined adjustment among the national GPS network (level A and B), the national GPS first class and secondary class net, and crust movement observation network of China(CMONOC) under ITRF.

The CGCS2000 geocentric coordinate is based on ITRF97, with the calendar of J2000.0. The positional accuracy is within

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3cm. there is high conformity between CGCS2000 and ITRF97 at J2000.0. The coordinate component accuracy of WGS-84 (1150) is within 1cm. Comparing CGCS2000 ellipsoid and WGS-84 ellipsoid, the longitude is same and the maximum latitude difference is nearly 0.11mm.

According to the comparison of the above we can draw the conclusion that the coordinate difference is at least under centimeters under the different framework. In order to analyse the influence resulted from the datum difference, this paper assumes that the coordinate of base station is precisely known and then turns the problem of datum difference to orbit error result from GPS ephemeris. The orbit error is about 12cm when finishing the transformation under the reality that the accuracy of CGCS2000 is within 3cm. The mean square error of a point of broadcast ephemeris is 5-7m and better than 5cm for precise ephemeris provided by IGS. So the key point is turned to the influence to RTK positioning result from orbit error.

2.3 Difference positioning principle

The most effective way for high precision GPS measurement is to use of high precision carrier phase observation data. RTK namely uses carrier phase observation value for positioning. The double difference observation equations is as follows

$$\begin{aligned}\Delta\nabla\phi_{ij}^{pq} &= \frac{1}{\lambda}(\rho_j^q-\rho_i^q-\rho_j^p+\rho_i^p)-\Delta\nabla N_{ij}^{pq}+\frac{1}{\lambda}\Delta\nabla d\rho_{ij}^{pq}\\ &+\frac{1}{\lambda}(-\Delta\nabla d_{ionij}^{\ \ pq}+\Delta\nabla d_{tropij}^{\ \ pq}+\Delta\nabla d_{mpij}^{\phi\ \ pq})+\frac{1}{\lambda}\Delta\nabla\xi_{\phi}^{pq}\end{aligned} \tag{1}$$

where

i,j =the station

p,q=satellite

$\Delta\nabla$ =double difference operator

ϕ =phase observation

ρ =distance between satellite and station computed by ephemeris and the approximate coordinate

$d\rho$ =satellite orbit error

N =fixed ambiguity

d_{ion} =ionosphere error

d_{trop} =troposphere error

d_{mp}^{ϕ} = multiply error

ξ_{ϕ} =carrier phase noise error

Among them, the ionosphere error and the troposphere error can be weakened well through corresponding error model, and the multipath error can be improved significantly by selecting good observation environment and GPS antenna (such as microstrip antenna) with its improved accuracy up to 95%.

define

$$\varepsilon=\frac{1}{\lambda}(-\Delta\nabla d_{ionij}^{\ \ pq}+\Delta\nabla d_{tropij}^{\ \ pq}+\Delta\nabla d_{mpij}^{\phi\ \ pq})+\frac{1}{\lambda}\Delta\nabla\xi_{\phi}^{pq} \tag{2}$$

Then the observation equation is like this

$$\Delta\nabla\phi_{ij}^{pq}=\frac{1}{\lambda}(\rho_j^q-\rho_i^q-\rho_j^p+\rho_i^p)-\Delta\nabla N_{ij}^{pq}+\frac{1}{\lambda}\Delta\nabla d\rho_{ij}^{pq}+\varepsilon \tag{3}$$

Supposing that the value of $\Delta\nabla N_{ij}^{pq}$ has been fixed by initialization, the expansion of series function at the approximate coordinate (X_i^0,Y_j^0,Z_j^0) is as follows

$$\Delta\nabla\tilde{\phi}_{ij}^{pq}=\frac{1}{\lambda}(\tilde{\rho}_j^q-\rho_i^q-\tilde{\rho}_j^p+\rho_i^p)-\Delta\nabla N_{ij}^{pq}+\frac{1}{\lambda}\Delta\nabla d\rho_{ij}^{pq}+\varepsilon \tag{4}$$

The linear equation after a double difference is

$$\Delta\nabla\phi_{ij}^{pq}=\Delta\nabla\tilde{\phi}_{ij}^{pq}+\frac{\partial\Delta\nabla\phi_{ij}^{pq}}{\partial\Delta\nabla X_j}dX_j+\frac{\partial\Delta\nabla\phi_{ij}^{pq}}{\partial\Delta\nabla Y_j}dY_j+\frac{\partial\Delta\nabla\phi_{ij}^{pq}}{\partial\Delta\nabla Z_j}dZ_j \tag{5}$$

where

$$\begin{aligned}\frac{\partial\Delta\nabla\phi_{ij}^{pq}}{\partial\Delta\nabla X_j} &= \frac{1}{\lambda}\left(\frac{X_j^0-X^p}{\tilde{\rho}_j^p}-\frac{X_j^0-X^q}{\tilde{\rho}_j^q}\right)\\ \rho_i^q &= [(X^q-X_i)^2+(Y^q-Y_i)^2+(Z^q-Z_i)^2]^{1/2}\end{aligned} \tag{6}$$

The similar formula for $\frac{\partial\Delta\nabla\phi_{ij}^{pq}}{\partial\Delta\nabla Y_j}$, $\frac{\partial\Delta\nabla\phi_{ij}^{pq}}{\partial\Delta\nabla Z_j}$, ρ_j^q, ρ_j^p, ρ_i^p .

If there are n satellites observed at the same epoch, one of the satellite was selected as the benchmark satellite, the error equations can be this matrix

$$\underset{(n-1)\times 1}{V}=\underset{(n-1)\times 3}{B}\cdot\underset{3\times 1}{\delta X}-l \tag{7}$$

where

$$\delta X=\begin{bmatrix}dX_j & dY_j & dZ_j\end{bmatrix}^T \tag{8}$$

$$l=\begin{pmatrix}\Delta\nabla\phi_{ij}^{12}-\Delta\nabla\tilde{\phi}_{ij}^{12}\\ \Delta\nabla\phi_{ij}^{13}-\Delta\nabla\tilde{\phi}_{ij}^{13}\\ \vdots\\ \Delta\nabla\phi_{ij}^{1n}-\Delta\nabla\tilde{\phi}_{ij}^{1n}\end{pmatrix} \tag{9}$$

B matrix can be denoted like this

$$B=\begin{pmatrix}\frac{\partial\Delta\nabla\phi_{ij}^{12}}{\partial\Delta\nabla X_j} & \frac{\partial\Delta\nabla\phi_{ij}^{12}}{\partial\Delta\nabla Y_j} & \frac{\partial\Delta\nabla\phi_{ij}^{12}}{\partial\Delta\nabla Z_j}\\ \frac{\partial\Delta\nabla\phi_{ij}^{13}}{\partial\Delta\nabla X_j} & \frac{\partial\Delta\nabla\phi_{ij}^{13}}{\partial\Delta\nabla Y_j} & \frac{\partial\Delta\nabla\phi_{ij}^{13}}{\partial\Delta\nabla Z_j}\\ & \vdots & \\ \frac{\partial\Delta\nabla\phi_{ij}^{1n}}{\partial\Delta\nabla X_j} & \frac{\partial\Delta\nabla\phi_{ij}^{1n}}{\partial\Delta\nabla Y_j} & \frac{\partial\Delta\nabla\phi_{ij}^{1n}}{\partial\Delta\nabla Z_j}\end{pmatrix} \tag{10}$$

When n satellites were observed on two stations, the corresponding weight matrix is defined as the follow formula at the same epoch.

$$P_{(n-1)(n-1)} = \frac{1}{2n\sigma^2}\begin{pmatrix} n-1 & -1 & \cdots & -1 \\ -1 & n-1 & \cdots & -1 \\ & & \ddots & \\ -1 & -1 & \cdots & n-1 \end{pmatrix} \quad (10)$$

Thus the least squares solution is

$$\delta X = (B^T PB)^{-1} B^T Pl \quad (11)$$

3. ANALYSIS ON ERROR

3.1 Analysis on orbit error

In order to further analyse the influence to positioning accuracy result from the orbit error $d\rho_{ij}^{pq}$, extracting the orbit error from the constant l.

In the network RTK, the coordinate of base station is precisely known. The orbit error can be decomposed into two components $\vec{\alpha}$, $\vec{\beta}$ as the figure shows. Where $\vec{\alpha}$ denotes the components from satellite to user j. $\vec{\beta}$ is at the another plane which is vertical to the plane(O)constituted by satellite and user j. In order to derivate a precise formula, two new orientations are built when decomposing $\vec{\beta}$ component. Where one component is on the intersection of plane(O)and another plane which is formed by satellite and baseline, the other component is on the plane(O)to $\vec{\beta}$.

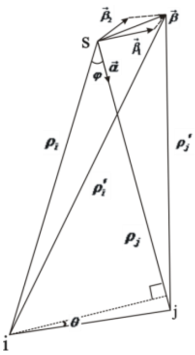

Figure.1 the influence to relative positioning result from orbit error

As shown above the figure, ρ_i, ρ_j, ρ_i', ρ_j' denote the distance without orbit error and distance with orbit error.

$$d\rho_j = \rho_j - \rho_j' = |\vec{\alpha}| \quad (12)$$

$$d\rho_i = \rho_i - \rho_i' = |\vec{\alpha}|\cos\varphi - |\vec{\beta}_1|\sin\varphi \quad (13)$$

A relationship can be get from the figure.1.

$$\rho_i' \sin\varphi = |\Delta\vec{S}|\cos\theta \quad (14)$$

So a useful formula was get as follows

$$d\rho_i - d\rho_j = -|\vec{\alpha}|(1-\cos\varphi) - \frac{|\vec{\beta}_1|}{\rho_i'}|\Delta\vec{S}|\cos\theta \quad (15)$$

define

ρ =20000km, and with the baseline length $|\Delta\vec{S}|$ =300km, then

$$\sin\varphi = \frac{|\Delta\vec{S}|\cos\theta}{\rho_i'} < \frac{|\Delta\vec{S}|}{\rho_i'} = \frac{300}{20000} \quad (16)$$

So we can get an inequation like this

$$|\vec{\alpha}|(1-\cos\varphi) < 1.28\times10^{-4}|\vec{\alpha}| \quad (17)$$

Then take the second term into consideration

$$\frac{|\vec{\beta}_1|}{\rho_i'}|\Delta\vec{S}|\cos\theta < \frac{|\vec{\beta}_1|}{\rho_i'}|\Delta\vec{S}| = 1.15\times10^{-2}|\vec{\beta}_1| \quad (18)$$

define $|\vec{\eta}| = \max\{|\vec{\alpha}|, |\vec{\beta}_1|\}$,So the orbit difference value

$$|d\rho_i - d\rho_j| < 1.17\times10^{-2}|\vec{\eta}| \quad (19)$$

Usually, the components of orbit error $|\vec{\alpha}|$, $|\vec{\beta}_1|$ are no more than 10m. The value is 0.23m when calculating the error with the 10m. So it must be taken into consideration for precision positioning.

Given the most unfavorable condition ($\theta=0$), the difference correction error result from orbit error and the length of baseline is shown in figure.2. Through the figure.2, a conclusion can be get is that the difference error correction increased with the increase of orbit error when the length of baseline is an invariant and increased with the baseline length when the orbit error is an invariant.

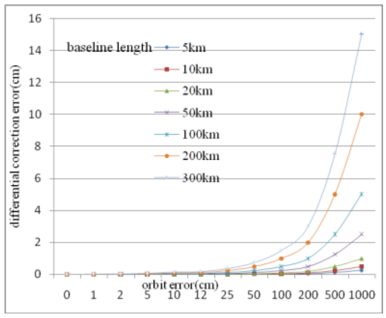

Figure.2 difference correction error result from the orbit error and baseline length

3.2 Error correction analysis result from the inconformity of datum

The influence to positioning was described in many papers when using the broadcast ephemeris or precision ephemeris.

The main aim of this paper is discussing the influence resulted from the datum inconformity. As mentioned before, a reasonable method is separating the orbit error transformed from the datum inconformity from the const l .

According the basic principle on difference positioning described before, we can know that the orbit error is included in the const l of error equation.

Supposing the precision ephemeris orbit error is ν_1 , the broadcast ephemeris orbit error is υ_1' , the orbit error result from the datum inconformity is υ_2 , and other comprehensive error is υ_3 .

For the precision ephemeris, define

$$\begin{cases} \upsilon_1+\upsilon_2=\frac{1}{\lambda}\Delta\nabla d\rho_{ij}^{pq} \\ \upsilon_3=\frac{1}{\lambda}(\tilde{\rho}_j^q-\rho_i^q-\tilde{\rho}_j^p+\rho_i^p)+\varepsilon \\ l_1=\Delta\nabla\phi_{ij}^{pq}+\Delta\nabla N_{ij}^{pq}-\upsilon_1-\upsilon_3 \\ l_2=-\upsilon_2 \end{cases} \tag{20}$$

So the error formula can be denoted like this

$$V=B\delta X-(l_1+l_2) \tag{21}$$

The least square solution is

$$\delta X=(B^TPB)^{-1}B^TPl_1+(B^TPB^{-1})B^TPl_2 \tag{22}$$

According to the same principle, there is a similar formula for broadcast ephemeris.

where

$$l_1'=\Delta\nabla\phi_{ij}^{pq}+\Delta\nabla N_{ij}^{pq}-\upsilon_1'-\upsilon_3=\Delta\nabla\phi_{ij}^{pq}+\Delta\nabla N_{ij}^{pq}-\upsilon_1-\upsilon_3-\upsilon_1'+\upsilon_1=l_1-(\upsilon_1'-\upsilon_1) \tag{23}$$

In the formula above, a masterly transformation is dividing broadcast ephemeris into two parts which are equivalent precision ephemeris and other part.

define

$$l_3=-(\upsilon_1'-\upsilon_1) \tag{24}$$

So the least square solution is

$$\delta X=(B^TPB)^{-1}B^TPl_1+(B^TPB)^{-1}B^TPl_2+(B^TPB)^{-1}B^TPl_3 \tag{25}$$

The meaning for each term in the formula above can be expressed respectively as follows

$(B^TPB)^{-1}B^TPl_1$: the comprehensive coordinate correction term when using precision ephemeris (equivalent precision ephemeris)

$(B^TPB)^{-1}B^TPl_2$: the coordinate correction resulted from the datum inconformity

$(B^TPB)^{-1}B^TPl_3$: the supererogatory coordinate correction relative to precision ephemeris when using broadcast ephemeris

There is only the first term when using precision ephemeris and the datum conformity. No matter how big the value of the first term, it is not what this paper care about. This paper focused on the second and third correction term. The difference value between broadcast ephemeris and precision ephemeris increased linearly with the growth of the baseline line in carrier phase difference positioning. The difference value was nearly 5cm on condition that the baseline length is 300km. (Jiao H S. 2009). The above analysis shows that the difference value is the third term in the formula before and the value is within 5cm.

According to the analysis before, the orbit error value resulted from datum inconformity was known $(|\vec{\beta}|<12cm)$, Precision ephemeris error $(|\vec{\eta}|=5cm)$ and broadcast ephemeris error $(|\vec{\eta}|=10m)$. A significant conclusion can be get from these values

$$(B^TPB)^{-1}B^TPl_2=1.2\times10^{-2}(B^TPB)^{-1}B^TPl_3 \tag{26}$$

Even in the worst circumstances that the baseline length is 300km, the coordinate correction is within 0.6mm. So the error resulted from the datum inconformity can be neglected completely.

4. CONCULUSION

According to the related discussion in this paper, the research presented in this paper has shown that the difference error correction resulted from the obit error increases linearly with the growth of orbit error under the condition that the distance between rover station and base station is unchanged in carrier phase difference positioning. At the same time, difference error correction increases linearly with the growth of the baseline when the orbit error is a const.

Based on the analysis in this paper, an important conclusion can be drawn is that no matter how high the precision requirement is, the influence of the datum conformity can be ignored completely. The accuracy of positioning when using precision ephemeris is not what this paper care about. In the RTK, many other methods usually are used to eliminate or weaken the error. So the error sprung from the datum inconformity can be ignored completely when using precision ephemeris or broadcast ephemeris and CGCS2000 coordinate for base station in RTK. Another conclusion can be drawn is that the error arise from the datum(CGCS2000,IITRF and WGS84) inconformity can be ignored for any positioning based on difference positioning.

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Acknowledgements and Appendix

This paper is funded by a 973 project which number is 2007CB310805. The author would like to thank my tutor -professor Hui,liu for his useful suggestions. Also I must thank my classmates for their stimulating discussions.