Regional Quasi-Geoid Determination in the Area of Poland

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Key words: geoid modelling, quasigeoid, astro-geodetic geoid, gravimetric quasigeoid, GPS/levelling quasigeoid, combined quasigeoid.

SUMMARY

The paper presents a short summary and more detail description of preliminary results of research on precise geoid (quasigeoid) model in Poland conducted in 2002-2005. After the introduction where general outline of the work is presented, in the next sections the procedures of quasigeoid determination using gravity, astro-geodetic, GPS/levelling data separately as well as in combined solution are described.

The developed quasigeoid models were intercompared. They were also compared with quasigeoid heights determined along the high precision, dense, 900 km long GPS/levelling traverse established for the project. The goodness of the fit of the obtained combined quasigeoid model, gravimetric quasigeoid model and astro-geodetic geoid model to the control traverse, measured with a standard deviation, equals to ±1.8 cm, ±2.1 cm, and ±7 cm, respectively.

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1. INTRODUCTION

Preliminary results of research on precise geoid (quasigeoid) model in Poland conducted in 2002-2005 in the framework of the project "Determination of a cm geoid model in Poland with the use of geodetic data, gravity data, astronomical data, geological data and satellite data" supported by the Polish Committee for Scientific Research are presented.

In the first step a qualitative and quantitative analysis of all available data, i.e. gravity data (terrestrial, sea-borne and air-borne), deflections of the vertical, GPS/levelling, altimetry, tide gauge, topographic data (DTM), and crust density was carried out. Over one million of terrestrial gravity point data acquired within numerous geological prospective projects conducted over last 50 years of 20th century was carefully analysed and transformed to recently adopted geodetic systems and gravity level (Krynski et al.,2005b). Also shipborne, and airborne gravity data as well as altimetry data for Southern Baltic Sea was collected and transformed to a uniform gravity system and geodetic datum (Jarmołowski, 2005).

Deflections of the vertical surveyed in the second half of last century were also analysed in terms of their usefulness for precise geiod modelling (Rogowski et al., 2005). Some new astronomical observations were also conducted for densification of the existing data as well as for control.

Quite extensive research was done on GPS/levelling data from EUVN network as well as from the networks that densify EUREF stations in Poland, all together consisting of nearly 1000 stations. Different algorithms of geoid height prediction based on GPS/levelling data with and without the use of gravity data were discussed (Krynski et al.,2005a).

Levelling data from 1st order vertical control network obtained during last levelling campaign (1998-2003) was analysed and compared with data from the previous campaign (1974-1982) (Łyszkowicz and Gajderowicz, 2005).

Quasigeoid models were tested with the use of different geopotential models, terrestrial gravity data and GPS/levelling heights at the GPS/levelling network sites. It was shown that the quality of the present solutions of GPS/levelling networks does not allow for quantifying the improvement of quasigeoid models due to the use of newly developed geopotential models (Krynski and Łyszkowicz, 2005).

Upper lithosphere density derived from geological survey in Poland was also analysed. The construction of database consisting of all data used for the project was created (Polechonska and Krolikowski, 2005). The dense, 900 km long GPS/levelling control traverse crossing Poland from south-western borders to northeastern ones was surveyed (Cisak and Figurski, 2005). Also some supplementary control gravimetric survey was conducted (Krynski et al., 2005b).

The last, prior to the recent project, gravimetric quasigeoid model for Poland was computed in 1997. Following quasigeoid models were based on gravity data transformed to a new gravity system POGK-99 and to ETRF89 reference frame with EGM96, GGM02S, GGM02S/EGM96, and GGM02C geopotential models (Łyszkowicz, 2005).

Final gravimetric quasigeoid model was computed from the revised point gravity data corrected for the effect of topography computed from high resolution DTM. The obtained quasigeoid model fitted to the national vertical datum is a useful tool in many geodetic and geophysical applications.

2. GRAVIMETRIC QUASIGEOID

The first gravimetric geoid model for the region of Central Europe, including Poland, was computed by (Tanni, 1949). The first gravimetric quasigeoid model for Poland was computed at Department of Planetary Geodesy in 1993, using the least squares collocation combined with the integral method (Łyszkowicz, 1993).

2.1 Computational method

Generally adopted strategy for computation of local geoid undulation N is composed of combination of three effects: global, regional and local, that are represented by the geopotential model GM, mean free-air gravity anomalies Δg_F, and topography (heights H), respectively.

$$N = N_{GM} + N_{\Delta g} + N_H \tag{1}$$

$$\Delta g = \Delta g_{FA} - \Delta g_{GM} - \Delta g_H \tag{2}$$

The term N_{GM} reflects the contribution of the GM coefficients, while $N_{\Delta g}$ represents the contribution of the residual mean free-air gravity anomalies after removing the effects of the *GM*, i.e. Δg_{GM}, and the terrain, Δg_H. N_H corresponds to the indirect effect of the terrain reduction on *N*. The contributions of the *GM* to *N* and Δg can be found in many publications.

$N_{\Delta g}$ is computed using Stokes' integral that in spherical approximation (Forsberg and Sideris, 1993) has the form

$$N_{\Delta g}(\varphi_p, \lambda_p) = \frac{R}{4\pi\gamma} \iint_E \Delta g\,(\varphi, \lambda)\; S(\psi)\; d\sigma \tag{3}$$

where E is the integration cup, R is the mean radius of the Earth, $S(\psi)$ is the spherical Stokes' kernel, and ψ is the spherical distance. That convolution integral can be evaluated in the frequency domain by the multi-band fast Fourier Transform. Proper zero padding (100%) was applied to the gridded data to eliminate the effects of circular convolution.

Since Stokes' formula is valid for Δg reduced to the geoid, all masses above the geoid must be mathematically shifted inside the geoid using terrain reductions. Thus the term Δg_H in (2) is the classical terrain correction.

The term N_H in equation (1) is called the indirect effect on the geoid, and accounts for the change of equipotential surface after the terrain reduction is applied to Δg. In case considered the geoid undulation term N_H was computed from digital terrain model using Helmert's condensation method (Wiechencharoen, 1982).

$$N_H = -\frac{\pi G \rho H^2}{\gamma} \tag{4}$$

Vertical control network of Poland was adjusted in both normal and orthometric height systems. Therefore the knowledge of both the geoid as well as quasigeoid is required. The computed geoid undulation N can thus be converted to the quasigeoid height ζ using the relation

$$\zeta = N - \frac{\Delta g_B}{\gamma} H \tag{5}$$

where Δg_B is simple Bouguer anomaly, γ is the mean normal gravity, and H is normal height.

2.2 Gravity data

Gravity data available at the area of interest is not uniform both in terms of quality and coverage. Terrestrial gravity data used for quasigeoid modelling consists of 2 × 2 km grid of free-air gravity anomalies generated from inhomogeneous set of point and mean gravity anomalies of different spatial resolution, acquired within last 50 years (Fig. 1). They differed in geodetic datums, gravity systems, normal gravity formulae, atmospheric corrections.

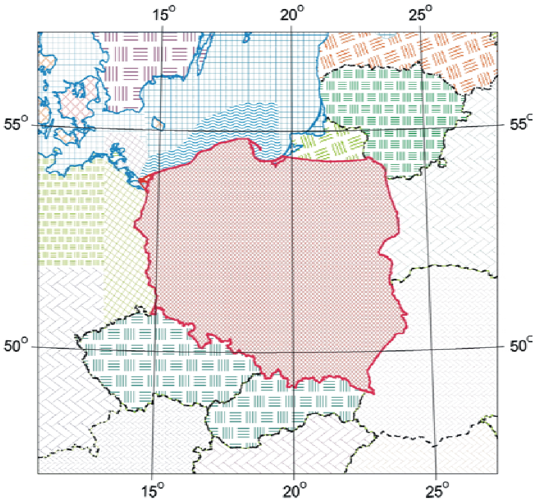

Fig. 1. Distribution of terrestrial and marine gravity data. Different data sets including those from different marine gravity surveys in Baltic Sea are distinguished with different colours and patterns.

Besides terrestrial gravity data some marine gravity data were available. They consist of shipborne data from the southern part of Baltic Sea, up to 100 km from the coastal line, acquired in 1978-1980 by former USSR research team. Marine gravity data acquired in coastal zone of Poland during the geophysical missions of Zaria and Turlejski vessels in 1971 and 1972 and recently released, were also considered.

Finally, marine gravity data from the Swedish coastal area of Southern Baltic Sea, acquired in 1999 by the Norwegian Hakoon Mosby vessel (provided by KMS) were also used.

All terrestrial and marine gravity data, including that from neighbouring countries has been transformed to ETRF89 and to POGK-99 gravity system (an official gravity system in Poland) (Krynski and Lyszkowicz, 2004). The 2 × 2 km grid obtained from that data was further used in numerical tests.

2.3 Geopotential models

The global geopotential model most frequently used for geoid modelling for last few years is the Earth Gravitational Model 1996 (EGM96). Gravity field recovery dedicated space missions that started in 2000 initiated a new era in global modelling of geopotential and furnished a series of geopotential models of growing resolution and of consecutively refined low-frequency components. From available models only models GGM02S, GGM02C and GGM02S/EGM96 were used in the study.

2.4 Practical computation

In the frame of the project on a cm geoid in Poland a few new gravimetric quasigeoid models were developed using remove-restore technique described in section 2.1.

The first quasigeoid model, called quasi04a, was based on the initial set of gravity anomalies, after correcting for geodetic datum and gravimetric system, and it was computed with the use of EGM96 geopotential model. The following quasigeoid models, quasi04b, quasi04c, and quasi04d, were based on the same set of gravity anomalies, but different geopotential models were used, namely GGM02S, GGM02S/EGM96, and GGM02C, respectively.

New set of gravity anomalies corrected for geodetic datum and gravimetric system at the level of point data was created at the beginning of 2005. That data set with geopotential model EGM96 was used to generate the new quasigeoid model, called quasi05a. After adding newly available gravity data in the eastern part of area of interest, the next quasigeoid model, quasi05b, was developed also with use of the EGM96. The last quasigeoid model, quasi05c, is based on the same gravity data as the previous one, but with use of GGM02S/EGM96 geopotential model.

The final model is given in a digital form on a regular grid 1.5' × 3.0' and is presented in a form of contour map.

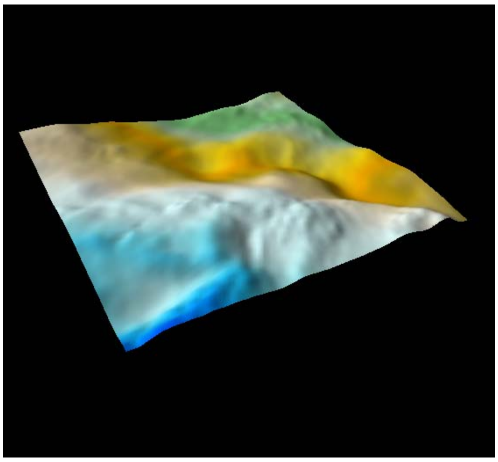

Fig. 2. Gravimetric quasigeoid for Poland.

2.5 Accuracy estimation

GPS/levelling-derived height anomalies precisely determined at the sites of the POLREF network were used for estimating the accuracy of computed gravimetric quasigeoid.

The POLREF network (Fig. 2) that is a densification of EUREF-POL92 network (11 Polish stations linked in 1993 to ETRF89) consists of 360 sites surveyed in three campaigns from July 1994 to May 1995 (Zielinski et al., 1997). Stations of POLREF network were linked to the national vertical control by spirit levelling (Kronstadt86 datum), with standard deviation of normal height equal to 1.0-1.5 cm, standard deviation of ellipsoidal height (GRS80 ellipsoid) 1.0-1.5 cm, and standard deviation of height anomaly equal to 2 cm (optimistic estimate).

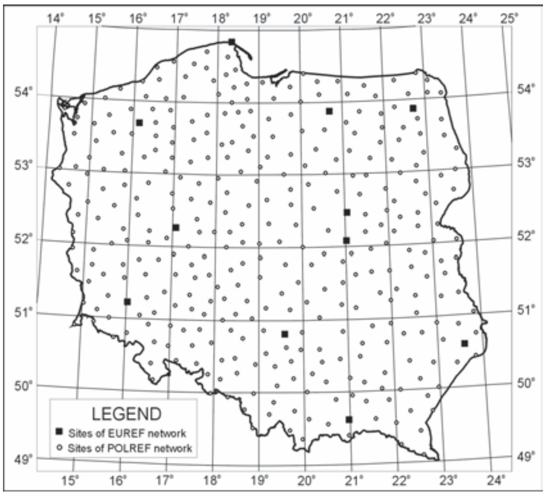

Fig. 3. Sites of EUREF-POL92 and POLREF networks

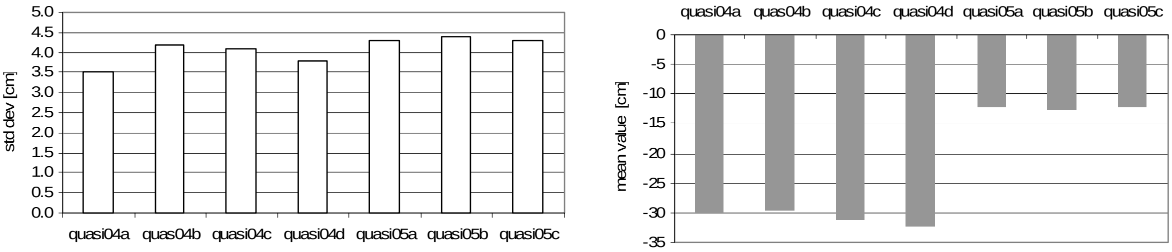

Fig. 4. Standard deviation (left), and mean value (right) of the differences $\Delta_i = N_i^{GPS/lev} - N_i^{grav}$ for the successive quasigeoid models

In order to evaluate the absolute accuracy of the computed models the differences

$$\Delta_i = N_i^{GPS/lev} - N_i^{grav} \tag{6}$$

were calculated at each point of the POLREF network. Mean value and standard deviation of those discrepancies for each quasigeoid model are shown in Fig. 4. Standard deviation remains almost the same; it slightly changes from 3.5 to 4.4 cm. The bias gets reduced, however, significatly from -30 cm to -12 cm after using refining gravity data from Poland.

2.6 The fit of the quasigeoid models to vertical datum

The discrepancies between computed gravimetric quasigeoid and quasigeoid heights derived from GPS/levelling data can be expressed as

$$l_i = \zeta_i^{gps/levll} - \zeta_i^{grav} \tag{7}$$

where $\zeta_i^{gps/levell}$ is a quasigeoid height derived from GPS and levelling and ζ_i^{grav} is a quasigeoid height from gravimetric model.

Modelling of the corrected surface begins by forming residuals (7), that can be split into trend *t*, signal *s*, and a noise *n* components in the least squares collocation model.

$$l_i = t_i + s_i + n_i \tag{8}$$

Next, since least squares collocation requires centered quantities (bias-free), the trend component is modelled by 3-parameter datum shift in the form

$$t_i = cos\,\varphi_i\, cos\,\lambda_i \Delta X + cos\,\varphi_i\, sin\,\lambda_i \Delta Y + sin\,\varphi_i \Delta Z \tag{9}$$

where φ_i, λ_i are geodetic latitude and longitude, and $\Delta X, \Delta Y, \Delta Z$ are datum shift constants. After computing the trend parameters, an empirical covariance function of the de-trended residuals (*l* - *t*) can be computed and modelled by a simple mathematical function. Markov covariance model in the form

$$cov(d) = C_o e^{-d/\alpha} \tag{10}$$

was applied, where *d* is the distance between points, C_o is the signal variance, and α is a value of an argument *d* for which $cov(\alpha) = \frac{1}{2} C_o$. After fixing the signal and the covariance model, the signal component can be computed at any station *P* from the formula

$$\hat{s}_P = \mathbf{C}_\mathbf{P}^\mathrm{T}(\mathbf{C}+\mathbf{D})^{-1}(\mathbf{L}-\mathbf{t}) \tag{11}$$

where $\mathbf{C}_\mathbf{P}$ is a signal covariance matrix between predicted signal and observations, $\mathbf{C}$ is a signal covariance matrix between observations, $\mathbf{D}$ is a covariance matrix of random measuring errors (noise) taken as diagonal and constant: $\mathbf{D} = \sigma_\mathbf{0}^2\mathbf{I}$, and (**L** - **t**) is a vector of de-trended observations with variance σ_o^2. Finally, the predicted signal and the trend component was added to the original gravimetric quasigeoid, to obtain the corrected surface in the form

$$\zeta_{new} = \zeta^{grav} + t + \hat{s} \tag{12}$$

In order to fit gravimetric quasigeoid model quasi05c to vertical datum, the discrepancies (7) at points of the POLREF'96 network were computed. Based upon the computed set of 330 discrepancies, datum shift components ΔX, ΔY, ΔZ were determined by using the least squares method. It yields $\Delta X = -0.301$ m, $\Delta Y = -0.275$ m, and $\Delta Z = 0.128$ m. The trend component was computed according to (9).

In the next step, the de-trended residuals and their empirical covariance function were computed. The signal variance of de-trended residuals equals to 12.67 cm^2 and the correlation distance is estimated as 0.58 degree.

The de-trended residual component was predicted on a 30' × 30' grid using (11) with the noise variance set up 4 cm^2 (see Łyszkowicz,1998, p. 274) and the results of the computations are displayed in Figure 5, which is very suggestive, although its interpretation is problematic.

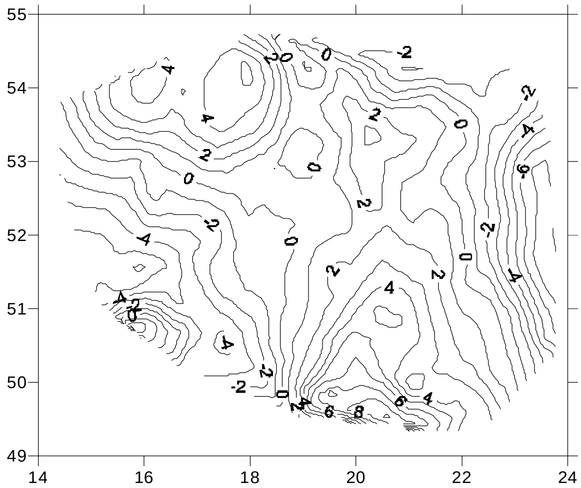

Fig. 5. Signal component (contour interval 1 cm)

Finally the corrections (12) were added to the quasi05c gravimetric model to produce new quasigeoid model quasi97c_corr model, that can be used in surveying practice. Estimation of accuracy of such corrected quasigeoid model will be given in section 6.

3. ASTRO-GEODETIC GEOID

In 1961 the first astro-gravimetric geoid for Poland was computed at the Institute of Geodesy and Cartography, Warsaw (Bokun, 1961). The absolute accuracy of this model was estimated on a few decimetres.

3.1 Deflections of the vertical data

Database for the deflections of the vertical in Poland consists of 171 astro-geodetic and 370 astro-gravimetric deflections of the vertical determined before 2001 (Rogowski et al., 2005). Distribution of the deflections of the vertical is shown in Figure 6.

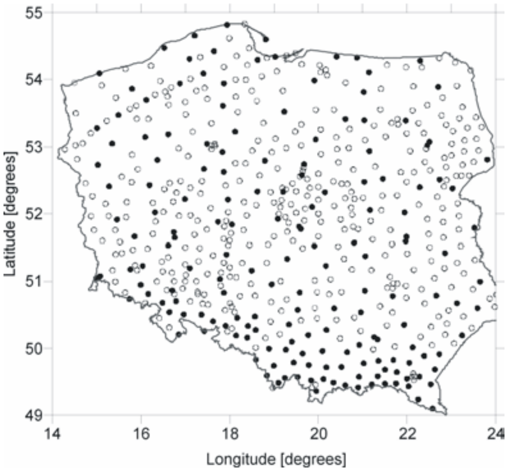

Fig. 6. Distribution of astro-geodetic (●) and astro-gravimetric points (○) in Poland determined before 2001

The standard deviation of astro-geodetic components was estimated as ±0.5", and astro-gravimetric as ±0.8" (ibid.). New astronomical observations at 29 points were performed with circumzenithal from May 2003 until July 2004 with standard error σ = ±0.3".

The original astro-geodetic and astro-gravimetric deflections of the vertical, that were measured nearly forty years ago, required a number of corrections. They were corrected due to star catalogue, polar motion, time system and geodetic datum. The corrected deflections of the vertical are referred to ETRF89, IERS pole and UT1.

3.2 Practical computation

The astro-geodetic geoid model was computed from 167 astro-geodetic archive deflections, 23 new astro-geodetic deflections, 7 astro-geodetic control deflections, and 364 astro-gravimetric archive deflections. The network formed of 561 points (including 188 astro-geodetic points) consists of 2099 lines (Fig. 7).

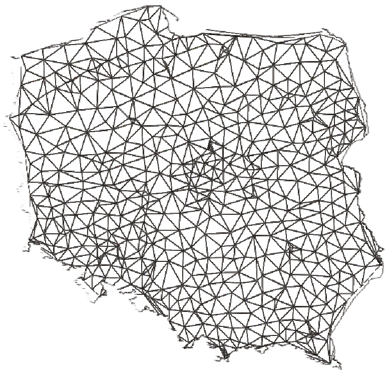

Fig. 7. Network created from astro-geodetic and astro-gravimetric points

Astronomical levelling was performed using the formula (Heiskanen and Moritz, 1967)

$$\Delta N_{1,2} = -\int_{1}^{2} (\xi \cos\alpha + \eta \sin\alpha + \delta\varepsilon)\, ds \tag{13}$$

where $\delta\varepsilon$ is the effect of the curvature of the plumb line.

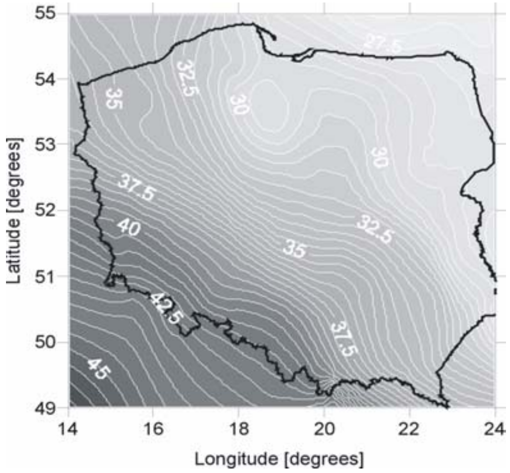

Fig. 8. Astro-geodetic geoid heights [m]

The network of geoid height differences ΔN has been adjusted with a parametric method using the GEOLAB software; geoid height at Borowa Gora was taken as fixed. Standard deviations of the components of astro-geodetic and astro-gravimetric deflections of the vertical were taken as 0.3" and 0.6", respectively, for calculating standard deviations of "observed" ΔN and then for determining weights. The empirical variance factor after adjustment was estimated as equal to 1.007. The correctness of the adjustment was verified with the χ^2 test. Standard deviations of the adjusted ΔN are within the range of (0 cm, 22.5 cm). Average standard deviation of the adjusted ΔN equals 6.4 cm (Rogowski et al., 2005). Practically, all points with standard deviations of adjusted geoid undulations exceeding 10 cm correspond to those with astro-gravimetric deflections of the vertical. The map of astro-geodetic geoid is given in Fig. 8.

3.3 Accuracy estimation

Astro-geodetic geoid model developed was compared with gravimetric geoid models. Geoid heights at the stations with deflections of the vertical determined were compared with the respective ones computed from the gravimetric quasigeoid model. Larger than a decimetre bias as well as tilt in meridional direction was observed. Standard deviation of the obtained differences reaches the level of 10 cm.

4. GPS/LEVELLING QUASIGEOID

4.1 Pure numerical GPS/levelling quasigeoid models

Two quasigeoid GPS/levelling models based on quasigeoid heights at the POLREF sites were developed (Krynski et al, 2005a). The first one applies the “kriging” model that is based on LS collocation with 4-order polynomial trend and a signal s

$$N(x, y) = a + bx + cy + dx^2 + exy + fy^2 + ... + s(x, y) \tag{14}$$

The second one involves the “minimum curvature” model with parameters determined using spline functions. Both quasigeoid models provided very similar results.

4.2 Numerical GPS/levelling quasigeoid model with support of gravity data

The “kriging” GPS/levelling quasigeoid model based on quasigeoid heights at the POLREF sites supported with mean 1' × 1' gravity anomalies Δg was further developed.

$$N(x, y) = \zeta(x, y) + a + bx + cy + dx^2 + exy + fy^2 + ... + s(x, y) \tag{15}$$

where

$$\zeta_{\Delta g}(X,Y) = \frac{1}{2\pi\gamma}\int_{-\infty}^{+\infty}\frac{\Delta g}{\sqrt{(x-X)^2+(y-Y)^2}}\,dxdy \tag{16}$$

4.3 Evaluation of GPS/levelling quasigeoid models at the EUVN sites

Pure numerical GPS/levelling quasigeoid model as well as the numerical GPS/levelling quasigeoid model with support of gravity data were evaluated at the EUVN sites. The statistics of the obtained differences ζ_{model} - ζ_{EUVN} are given in Table 1.

Table 1

Model	Mean	Std dev.	Min	Max
pure numerical	-0.034	0.048	-0.213	0.103
with gravity	-0.031	0.029	-0.124	0.021

5. COMBINED GEOID

5.1 Quasigeoid model combining GPS/levelling with gravity and topography data

A combined GPS/levelling/gravity qasigeoid model was developed (Osada et al., 2005). It uses LS collocation for fitting the quasigeoid model $\zeta_{GM} + \zeta_{\delta g} + \zeta_{G_1}$ to height anomalies $\zeta_{GPS/lev}$ obtained at GPS/levelling sites with simultaneous determination of model parameters. $\zeta = \zeta_{GM} + \zeta_{\delta g} + \zeta_{G_1} + t + s$ (17) $\zeta_{GM} = \frac{W_P^{GM} - U_P}{\gamma_Q}$ (18)is the global geopotential model component of the height anomaly,

$$\zeta_{\delta g} = \frac{1}{2\pi\gamma} \iint_{\sigma} \frac{\delta g}{r} d\sigma \tag{19}$$

is the gravimetric component of the height anomaly,

$$\zeta_{G_1} = \frac{1}{2\pi\gamma} \iint_{\sigma} \frac{G_1}{r} d\sigma \tag{20}$$

is its terrain component with

$$G_1 = \frac{1}{2\pi} \iint_{\sigma} \frac{H - H_P}{r^3} \Delta g d\sigma \tag{21}$$

and t is the trend expressed by 4-order polynomial function (14). The method is quite efficient and robust for detection outliers.

5.2 Evaluation of the combined quasigeoid model

The combined GPS/levelling/gravity quasigeoid model developed was evaluated at GPS/levelling sites of both POLREF and EUVN networks. The statistics of the obtained differences ζ_{model} - $\zeta_{POLREF/EUVN}$ [m] are given in Table 2.

Table 2

Reference sites	Mean	Std dev.	Min	Max
POLREF	0.005	0.007	-0.013	0.030
EUVN	-0.005	0.006	-0.023	0.012

6. COMPARISON OF QUASIGEOID MODELS

To assess the accuracy of computed quasigeoid models a precise GPS/leveling control traverse at levelling benchmarks was established, surveyed and adjusted. The traverse contains nearly 200 points 3–5 km apart. Observations were conducted twice in 4–24h sessions and were linked to permanent stations. Estimated accuracy of the height components after adjustment of the traverse is ±1 cm.

After close inspection, some points of the traverse were rejected and finally 187 points were used to estimate the precision of the computed quasigeoid models quasi04a, quasi04b, quasi04c, quasi04c, quasi04d, quasi05a, quasi05b, quasi05c_corr and quasi_combined. The results of comparison are shown in Figure 9. Quasigeoid models quasi04a and quasi04b exhibit the largest systematic shift with respect to the heights at the traverse stations, while the biases of quasi04c, quasi04b computed from the revised gravity data are twice smaller. The best agreement with the traverse demonstrate quasi_combined and quasi05c_corr models, that are fitted to Polish vertical datum.

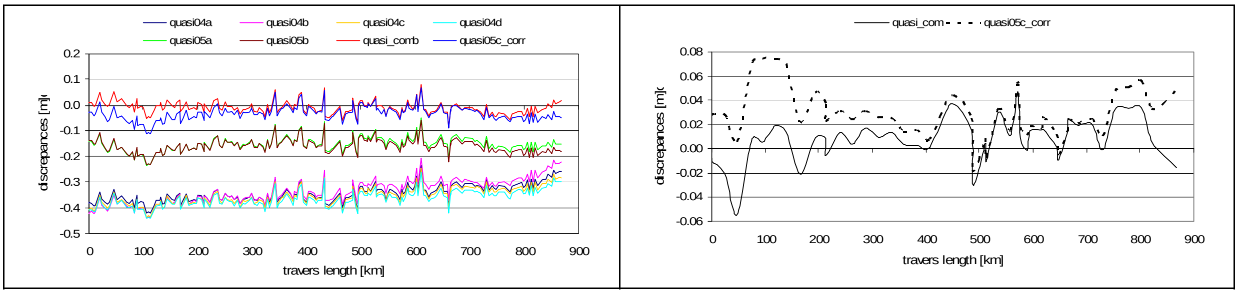

Fig. 9. Comparison of quasigeoid models in respect to GPS/levelling heights of control traverse. All travese points (left), only points observed in 24 hour sessions (right)

There are no doubts that the heights of the points that were observed in 24h session are more accurate. Therefore, the last quasigeoid models: quasi05c_corr and quasi_combined were evaluated with the use of those points of the traverse only. The result is shown in Figure 9. Both models show the same feature, although quasi_combined is slightly closer to the traverse than quasi05_corr. It is better visible when comparing mean values and standard deviations of the obtained discrepancies (Table 3).

Table 3 Statistics of comparison

	Quasi_com	Quasi05c_corr
Mean	0.007	0.027
Std dev	0.018	0.021

Also the astro-gravimetric quasigeoid model interpolated to the points of the control traverse was compared with the quasi_combined one (Fig. 10).

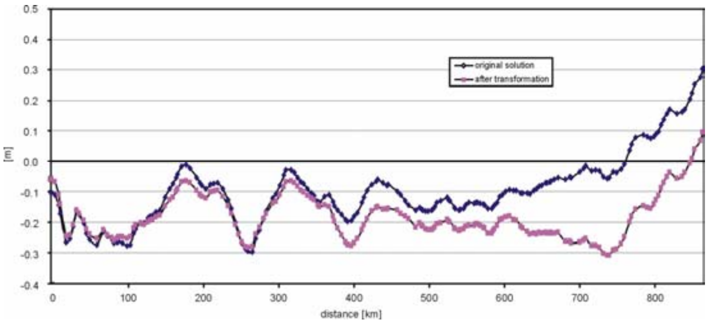

Fig. 10. Comparison of astro-gravimetric quasigeoid model with the quasi_combined one along the control traverse

Standard deviation of original solution (blue curve) equals to 11 cm, while after transformation it lowers to 7 cm. Both bias and tilt of the astro-gravimetric geoid model is visible in Figure 10. The results shown reflect the errors of interpolation that might in some regions be quite large due to sparse and irregular distribution of astro-geodetic deflections of the vertical.

7. SUMMARY AND CONCLUSIONS

The results of the project on modelling a centimetre geoid in Poland with the use of geodetic, gravimetric, astronomic, geological and satellite data can be considered fully successful. All available data has been gathered and extensively qualitatively and quantitatively analysed. The methods of transforming data acquired in different epochs using different techniques to unified standards, scales and reference systems have been developed. The data was archived and appropriate databases were developed. Particularly, the extensive research was done on over a million of gravity data from the area of Poland including Southern Baltic Sea, acquired within last 50 years, that have became for the first time available for gravity field modelling.

New gravimetric quasigeoid models based on gravity data transformed to a new gravity system POGK-99 and to ETRF89 reference frame with EGM96, GGM02S, GGM02S/EGM96, and GGM02C geopotential models were computed. Finally gravimetric quasigeoid model was computed from the revised point gravity data corrected for the effect of topography computed from high resolution DTM. The obtained quasigeoid model fitted to the national vertical datum is a precise and useful tool in many geodetic and geophysical applications

Also new astro-geodetic geoid model based on deflections of the vertical surveyed in the second half of last century and on some new astronomical observations was computed.

Research was also successfully completed on the combined quasigeoid model based on gravity, GPS/levelling and topographic data. Analytical robust method of quasigeoid modelling developed proved its resistance to the outlying measurements. It also allows for the detection of the outliers what was verified at the sites of the POLREF network. It is an original achievement that can be used for generating quasigeoid best fitting to the

GPS/levelling sites of the national control, and as such can be applied in practical surveying and precise navigation.

For verification of quasigeoid models derived as well as for estimation of their accuracy and evaluation of interpolation algorithms used for application of GPS/levelling quasigeoid a 868 km long control GPS/levelling traverse across the country has been established. Observation strategy developed and processing methodology applied ensure the accuracy of quasigeoid heights at traverse points at a centimetre level.

Data collected, its reliable accuracy evaluation, developed methods and computing strategies as well as experience gained reflect high potentiality for further research on developing precise quasigeoid models in Poland, simultaneously indicating the most important fields of investigations and needs for acquiring complementary quality data.

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