GPS Levelling Without Geoid in Egypt Applied to Borg El-Arab City

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Key words: GPS, EGM96, GRACE, Levelling, Vertical Control

SUMMARY

Global Positioning Sytem, GPS, has been used extensively in most of Engineering applications. This system offers fast and accurate positioning compared to the conventional methods. The GPS ellipsoidal heights are not suitable as vertical control in engineering applications. On the other hand, the use of careful GPS survey procedures coupled with high-resolution geoid models, to obtain orthometric heights is known by GPS levelling.

In this paper, GPS levelling technique is presented to densify local Benchmarks instead of time consuming and costly traditional technique i.e, levelling. The areas and distances in Egypt suitable to neglect the presence of geoid are investigated and determined based on the allowable levelling error. This sutdy is done using the geoid resulting from the recent global geopotential models i.e. GRACE, and GPM98A. The areas with gentle slope geoid undulations are focused on. The technique is then applied to Borg El-Arab city. The results are compared to the levels determined by levelling technique. The errors are determined in case with no geoid and then with introducing geoid fromGPM98A, EGM96, and GRACE models. The errors are presented and concluded. The concept of geometric geoid or corrector surface was introduced the corrector the observed ellipsoidal height and convert it to orthometric height. Four models were tested as corrector surfaces to choose only one. The results after using corrector surface indicate that the GPS levelling can replace the traditional levelling.

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1. INTRODUCTION

The determination of orthometric heights by traditional techniques, such as spirit levelling, is known to be a difficult task (Vergos, G.S. and Sideris, M.G., 2002). This is especially evident in developing countries like Egypt where the establishment of a levelling network covering all parts of the country would be impractical from the financial point of view. Moreover levelling over areas with rough terrain is very tough and time consuming. In addition, the levelling network had been established many years ago and significant number of benchmarks has been destroyed and some of them has been exposed to unknown vertical deformation. Accordingly, the problems of densfying, replacing, monitoring, and maintaining of the levelling network in Egypt have constrained the development in different areas, and initiated the necessity of using other techniques.

On the other hand, heights obtained by GPS are above an ellipsoid and are fundamentally different from traditionally obtained heights which are given with respect to the geoid. Mathematically, there is a simple relation between the two concepts and may be given by:

$$H = h - N \tag{1.1}$$

In practice, the expression (Eq. 1.1) reflects the possibility of GPS levelling, because it states that if the geoidal height N is known, the orthometric height H can be obtained from ellipsoidal height h determined by GPS. Obtaining orthometric heights this way, could in certain circumstances, depending on the required accuracy, replace conventional spirit levelling and thus make the levelling procedure cheaper and faster which is necessary for both practical surveying and scientific applications (Martensson, S.G., 2002).

The accuracy of classical levelling is a complex subject in itself, and for engineering applications it may be taken 6mm√km (Rummel, R., 1992). In order to satisfy this accuracy using GPS levelling, there are two different things which we have to take into account: the accuracy of the GPS itself and the accuracy of the geoid model we need to transform heights above the ellipsoid to orthometric. Differential GPS measurements, which are used for ellipsoidal height determination, are now known for their high-accuracy because of the availability of a greater number of satellites, more accurate satellite orbits, full wavelength dual-frequency carrier phase data, improved antenna designs, and improved data processing techniques (Zilkoski, D.B., et al 2001).

The critical part of this method is the geoidal height, which normally is obtained with a lower accuracy than the ellipsoidal height and thus affecting the accuracy of the orthometric height. The geoidal height can be determined using the global geopotential models alone or from global models combined with the local gravity data i.e. gravimetric geoid model. If the available gravimetric geoid model does not meet the required accuracy, the problem can be overcome by creating a local model of the geoid, a geometric geoid model, by measuring h_i (with GPS equipment) on several points (i) with known orthometric heights H_i

In Egypt, it is urgently recommended to use the GPS levelling technique as an alternative potential to the classic geometric levelling, but unfortunately, the lack of dense gravity data has restricted the production of accurate geoid model. The only trails for gravimetric geoid determination in Egypt have been done through the combination of global models with the available scattered gravity data. The results are not accurate enough to support the GPS levelling technique. To produce an accurate local geoid for Egypt, it has been recommended to measure more gravity data spaced by 5km to cover the entire territory. This would be impractical financially and result in inaccessible points due to rough trerrain of many areas in Egypt. Moreover, investigating the current situation for gravity data in Egypt, it can be noticed that the suitable areas to add more gravity data (accessible areas)are the same areas covered with levelling network. Finally, the possible way to have an accurate geoid model would be a local geoid model at areas with available dense gravity data.

In this paper, it is suggested to define the areas with gentle geoid slope which reflect small geoidal heights variation within the allowable levelling accuracy or even more, in case no other solution exists. In this case, the geoidal heights variation or difference would be considered as errors, i.e., allowable for engineering applications. The recent global model i.e. GRACE complete to degree and order 200 was used in this to determine the accurate long wavelength contribution of geoid model in Egypt. GPM98A Model (Wenzel , G., 1998) complete to degree and order 1800 was also used to estimate the short wave length contribution of the geoid model. The two models were merged to produce global geoid model for Egypt.

On the other hand, Borg El-Arab city is one of the new urban communities cities. It has been constructed near northern coastal zone of Egypt. The area of the city is 50000 fed, extended around 27 km in east west direction and around 8 km in north south direction. It was required to produce topographic maps for the city and to establish benchmarks tied to national datum and covered the city with spacing around 2500 m among them. The estimated time and cost to perform levelling procedure were beyond the expected and designed for this project. Fortunately, Borg El-Arab city has an old benchmarks distributed around and inside the area (48 points) and tied to national datum. The GPS levelling technique was used to determine orthometric heights for new established benchmarks. The benchmarks were observed by dual frequency receivers. The geoidal heights for the old benchmarks with known orthometric heights were estimated. Also, the concept of no geoid was examined and concluded. The global geoid models i.e. GPM98a, EGM96, GRACE were used to determine the geoidal heights and the results were investigated. Four Corrector models were tested to enhance the orthometric heights obtained by GPS levelling supported by geometric geoid model. The results will be illustrated.

2. GEOIDAL HEIGHTS COMPUTED FROM GLOBAL GEOID MODELS

Global geoid models represent the long wavelength part of the gravity field and are obtained from global geopotential solutions which are given as a set of spherical harmonic coefficients (Sideris et al. 1992). Different datasets are used to determine these coefficients ranging from satellite observations, which give the so-called satellite-only solutions, to models which incorporate satellite altimetry and surface gravity data in the previous ones, thus usually containing more coefficients. To compute a geoidal height value N_{GM} in spherical approximation and from such a set of spherical harmonic coefficients we use the following formula (Rapp, R.H., 1982):

$$N_{GM} = \frac{T}{\gamma} = \frac{GM}{r\gamma} \int_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \int_{m=0}^{n} (\overline{C}^*_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda)\, \overline{P}_{nm}(\cos\theta) \tag{2.1}$$

where: GM is the geocentric gravitational constant; r is the geocentric radius; θ is the geocentric colatitude; λ is the geocentric longitude; P_{nm} are the fully normalized associated Legendre functions; C_{nm}, S_{nm} are the fully normalized potential coefficients of the Earth; and a is the semi-major axis of the defined Earth ellipsoid. Three global models were used in this study i.e. GPM98A , GRACE, and EGM96 models.

The ultra high degree geopotential model GPM98A was computed by Wenzel from world wide sets of 5' x 5' mean free air gravity anomalies. The geopotential model have a maximum degree and order of 1800 (3.243 million coefficients) which corresponds to about 10 km spatial resolution. The coefficients of GPM98A and B up to degree 20 have been taken from geopotential model EGM96; the coefficients from degree 21 to degree 1800 have been computed from the 5' x 5' mean free air gravity anomalies in an iterative procedure. For the start of the iteration, geopotential model EGM96 has been used. The ultra high degree geopotential models GPM98A has achieved r.m.s. discrepancies of about 5 cm (Wenzel, G., 1998).

The GRACE gravity model GGM01C complete to degree and order 200 was released on July 2003. This model is a combination among Satellite gravity model only GGM01S and surface gravity data , altimetric sea surface heights. GGM01S was estimated with 111 days (spanning April through November of 2002) of GRACE K-band range rate, attitude, and accelerometer data (Tapely,B.D., and Reigber, C., 2003) .

On the other hand, EGM96 is consisting of spherical harmonic coefficients complete to degree and order 360. It is a composite solution, consisting of: (1) a combination solution to degree and order 70; (2) a block diagonal solution from degree 71 to 359; and (3) the quadrature solution at degree 360. This model is the result of a collaboration between the National Imagery and Mapping Agency, the NASA Goddard Space Flight Center, and the Ohio State University (NGA, 2004).

3. GPS LEVELLING WITHOUT GEOID

Recall, One of the main goals of a common treatment of levelling, GPS and the geoid is to substitute expensive levelling through cheaper GPS measurements at least in regions where not the highest accuracy is necessary. Orthometric heights determination by GPS levelling requires accurate positioning by GPS and an accurate geoid model. Unfortunately, in Egypt the trails to determine local accurate model are restricted due to lacking of gravity data. On the other hand, different development projects have been taken place in Egypt. The common problem for these projects is to find out the possible ways to tie the projects to national vertical datum and densify the existing benchmarks. In this context, it is suggested to find out the areas in Egypt with flat geoid or with minimum geoidal height slope. Under such condition, it is possible to consider the case of no geoid exists. In this case, the geoidal height variation should be within the allowable levelling error i.e. 6mm√km or even more, depends on the possible solutions for orthometric height problem.

The geoidal heights for Egypt were determined from global models. GRACE model is known to be accurate to determine the long wavelengths of geoid, but it is unsuitable for the short wavelengths contribution. Accordingly, The GPM98A model was used to compute the short wavelengths contributions. GPM98A model is complete to degree and order 1800 and capable to resolve wavelengths up to 10 km. GRACE model is complete to degree and order 200. The two models were combine to enhance the long wavelengths of GRACE model and to add the short wavelengths from GPM98A. A contour map with contour interval 2 cm is prepared to show the variation of geoidal heights. Figure 3.1 represents the geoidal heights at the areas where the geoid slops seems to be nearly flat. The figure indicates the geoidal

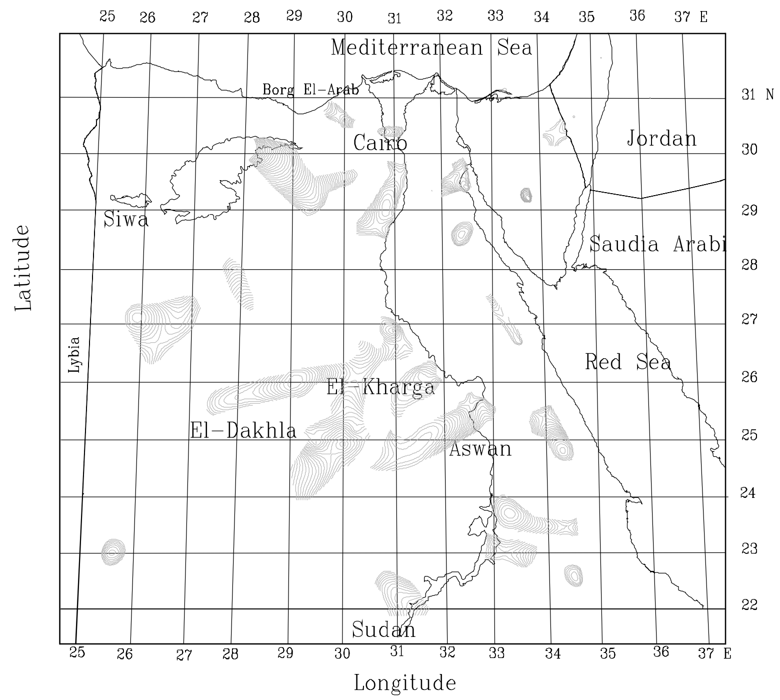

Figure 3.1 Areas with Gentle geoid slope In Egypt, C.I. 2 cm

heights with contour interval 2 cm. The distances between contour lines are chosen to be within or more than 5 km.

Most of the areas are located in the western desert. The areas are in El-Wadi El-Gadid, Aswan, and Al-Fayom Governorates. The area around the Cairo –Alexandria desert road also indicates gentle geoid slope. Also the area north of Cairo and the area at the eastern boundaries are with nearly flat geoid. Accordingly, the concept of using GPS levelling without geoid can be applied at these area depends on the accuracy required and the available levelling information. In other words, one has to collect all the information about the new projects and decides the method of obtaining orthometric heights based on the required accuracy.

4. TESTING GEOID FROM GLOBAL MODELS AND LOCAL "GEOMETRIC" GEOID TO SUPPORT GPS LEVELLING IN BORG EL-ARAB CITY

The main Objective for the survey tasks at Borg El-Arab city was to establish an accurate network of control points to be a base for map production and future planning and construction. The horizontal control was decided to be performed using GPS (dual frequency receivers). The vertical Control was suggested to be done using the levelling technique. The estimated time and cost to finish such task was out of the planned issues. Accordingly, it was decided to test the possibility of replacing the levelling by using GPS levelling. Based on the fact that there was no accurate geoid model for this area, we decided to test the available global geoid models.

On the other hand, the city is surrounded by border points tied to the national vertical datum. A group of points were also found inside the City. These points (48) were observed using GPS dual Receivers and the ellipsoidal heights were determined. The global geopotential models for GRACE complete to degree and order 200, EGM96 complete to degree and order 360, and GPM98A complete to degree and order 1800 were used to compute the geoidal heights were for each point. The orthometric heights for the 48 points were estimated using GPS levelling basis. The errors in the estimated GPS levelling orthometric heights compared to levelling orthometric heights were estimated. The statistics of the resulting errors are shown in Table 1. The errors also were estimated for case of no geoid. The results in Table 1 indicate that the errors range between -12 cm to 18 cm . The standard deviation estimated for the results indicates that the GPM98A model gives minimum errors compared to the other models.

The errors are also presented as contour map for each case of using global geoid model. Figures from 4.1 to 4.4 show the resulting errors as contour lines with contour interval 1 cm. The figures do not show fixed pattern for the errors i.e. no specific increase or decrease in one direction. The resulting errors are in the order of some centimetres and do not seem appropriate to replace classical levelling by GPS levelling, but with refined observation and data analysis techniques it certainly will.

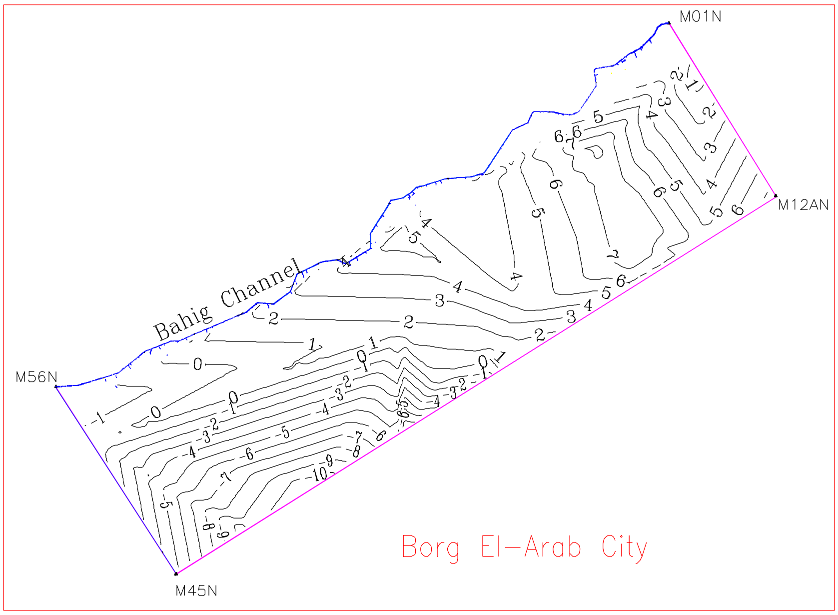

Figure 4.1 Error inOrthometric Heigts Determined by GPS Levelling Based on No Geoid Compared to Levelling Orthometric Heights for 48 Points

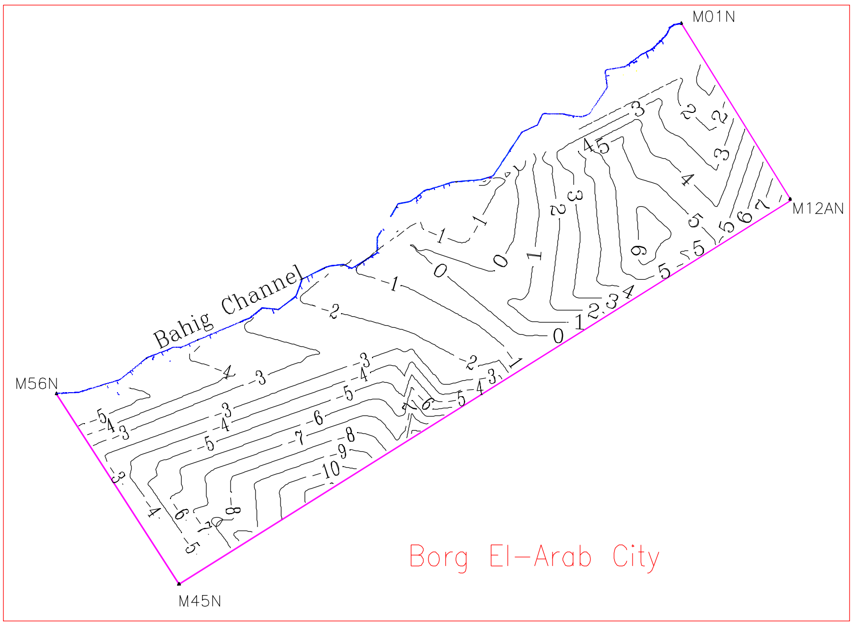

Figure 4.2 Error in Orthometric Heigts Determined by GPS Levelling Using Global Model GRACE Compared to Levelling Orthometric Heights for 48 Points

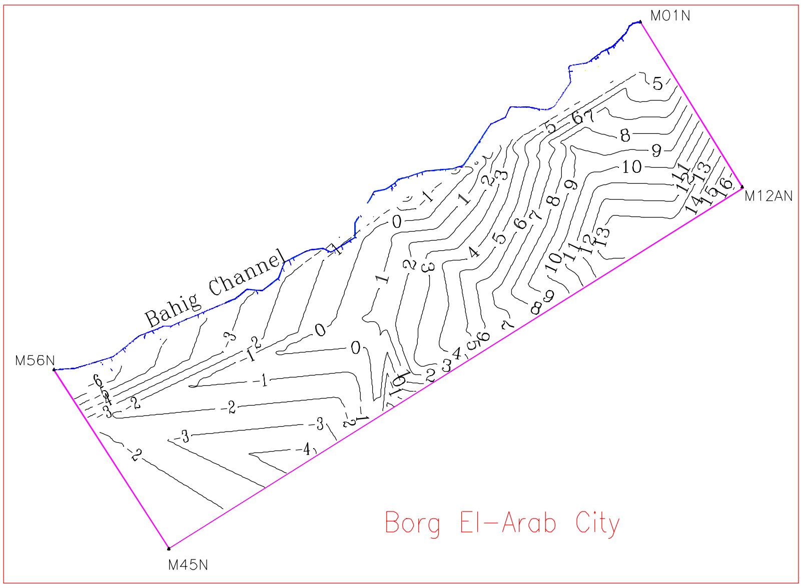

Figure 4.3 Error inOrthometric Heigts Determined by GPS Levelling Using Global Model EGM96 Compared to Levelling Orthometric Heights for 48 Points

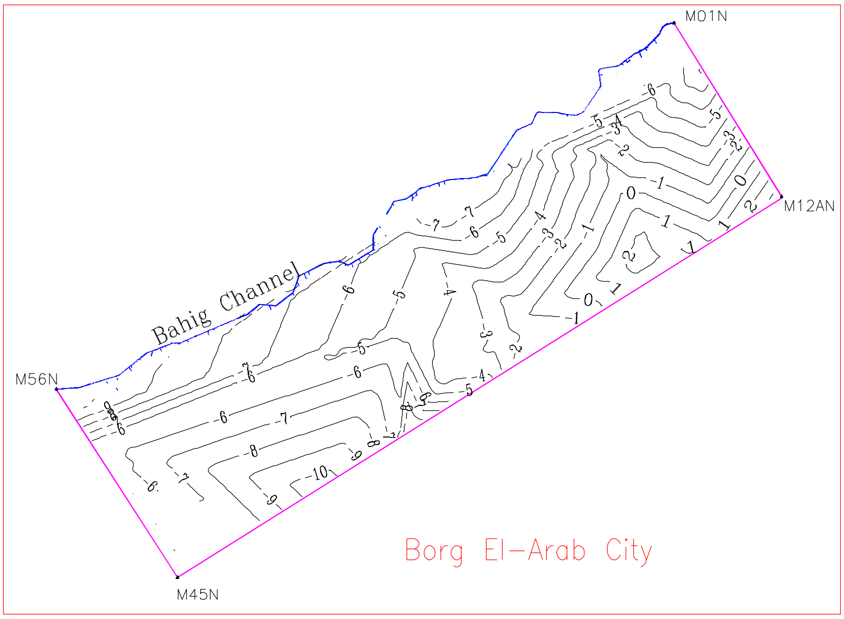

Figure 4.4 Error inOrthometric Heigts Determined by GPS Levelling Using Global Model GPM98A Compared to Levelling Orthometric Heights for 48 Points

Table 4.1 Comparison of Errors in Orthometric Heigts Determined by GPS Levelling Based on Global Models and Levelling Orthometric Heights for 48 Points at Borg El-Arab City

Geoid Model	Min. (cm)	Max. (cm)	Mean (cm)	Std (cm)
No Geoid	-11.2	8.6	2.2	5.4
GRACE	-11.7	8.6	-.1	5
EGM96	-7.4	17.6	4.7	5.9
GPM98A	-10.7	3.6	-4.7	3.9

Accordingly, the concept of local "geometric" geoid model or corrector surface is suggested here using the geoidal heights computed at 48 points with known orthometric heights. Only 38 points were used to develop a corrector surface. The development of corrector surfaces aims basically at providing GPS users with an optimal transformation model between ellipsoidal heights h and orthometric heights H with respect to a given levelling datum (Kotsakis,C, Sideris,M.G. 1999). The results of this modelling of corrector surface can be used to correct any estimated orthometric height resulted from GSP levelling without geoid. The rest 10 points were used to test the various used models. Four models were used to choose the most appropriate corrector surface. The triangulation with linear interpolation, polynomial regression, kriging, and inverse distance methods were used to model the errors or to define the corrector model. Details of the mathematical models can be found in (Soycan,M, and Soycan, A., 2003, Kotsakis,C, Sideris,M.G. 1999). The SURFER software package version8.0 was used in testing the four models. The statistics of the residuals for the ten points are shown in Table 4.2 the Kriging method proves to be the best corrector surface. The residuals are in the range of one centimetre. Accordingly, the method of GPS levelling without geoid has proven to be effective by using the concept of producing local geometric geoid or producing corrector surface.

Table 4.2 Comparison of Some Methods for Modelling of Errors in Orthometric Heights obtained from GPS Levelling in Case of No Geoid

Error Modelling Method	Min. (cm)	Max. (cm)	Mean (cm)	Std (cm)
Tringulation with linear Interpolation	-0.33	1.82	0.42	0.76
Polynomial Regression	-0.91	4.10	1.93	1.53
Kriging	-0.44	1.08	0.26	0.59
Inverse Distance	-0.74	1.73	0.51	0.74

5. CONCLUSIONS

GPS levelling technique has the potential to replace the classical levelling in orthometric height determination. By this technique, it would be possible to determine the orthometric heights in less time and cost than the traditional technique. The accuracy of orthometric heights resulting from GPS levelling depends mainly on the accuracy of used Differential GPS, where the use of dual frequency receivers is recommended. It also depends on the accuracy of used geoid model. In case of no accurate geoid available, the use of such technique would not be that forward. In this paper, the concept of using no geoid were introduced. Based on merging the global geopotential model GPM98A complete to degree and order 1800 and grace Model complete to degree and order 200 the geoidal heights for

Egypt were computed. The areas with gentle geoid slopes were determined. In these areas GPS levelling technique may be used without the need for considering geoid information. The criteria used in such classification was to define the area with geoidal variation do not exceed 2cm in distances more than 5km. The areas are mainly located in Western desert , especially at El-Wadi El-Gadid governorate.

The GPS levelling technique was also applied at Borg El-Arab city. Based on the existing Vertical control points (48 points). The global geoid models were tested to support GPS levelling in producing orthometric heights. The GPM98A model was proven to be the best global model as long as it produces small errors compared to the others. The global models do not improve the errors of orthometric height resulted from GSP levelling. The concept of using geometric geoid or corrector surface is suggested here. Four models i.e. the triangulation with linear interpolation, polynomial regression, kriging, and inverse distance methods were tested as corrector surfaces . The kriging method produces minimum residuals compared to the others. The residuals after using corrector surface indicate that the GPS levelling is capable to replace the traditional levelling.

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BIOGRAPHICAL NOTES

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