Correction Model to Rectify Distorted Co-ordinate Systems

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SUMMARY

Sweden consists of some 300 municipalities, all of them more or less using their own geodetic control networks. The local systems are usually old and not strongly linked to the national co-ordinate systems. Lantmäteriet is not able to force the local authorities to adopt the national co-ordinate system also locally, why more or less every local authority has its own system. Because of the way that the systems have been developed, the local systems are often distorted. When measuring using the nearest point as reference these distortions are usually not influencing the measurements but when using GNSS as your measuring tool, these distortions are important to handle. Local authorities are today using GNSS more and more, especially our SWEPOS Network RTK service. Therefore it is more and more important to have control of the distortions of the local system.

Most of the local authorities are now changing to use the national reference system also locally to make use of GNSS in their own organisation as well as harmonising their data with the existing regional and national data. A correction model based on the residuals existing after the transformation between the national reference frame SWEREF 99 and the local co-ordinate system improves the result for the users. A model can also be used nationally, of course.

Lantmäteriet has developed some tools to analyse the residuals and to create the correction model. These models can then be used either when the local authority is changing to a SWEREF 99-based co-ordinate system or when distorting GNSS measurements so that they fit to the existing co-ordinate system.

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1. INTRODUCTION

Sweden consists of some 300 municipalities, all of them more or less using their own unique co-ordinate system. Most of them are now changing to use the national reference system also locally to make use of GNSS in their own organisation as well as harmonising their data with the existing regional and national data. A correction model based on the residuals existing after the transformation improves the result for the users. A model can also be used nationally, of course.

2. BACKGROUND

2.1 Local Co-ordinate Systems versus National Co-ordinate Systems

The first control networks for municipalities were established in the beginning of the last century. Most of them were in a very weak way connected to the national network prevailing at that time in Sweden. Since then control networks have been established in almost every urban area. Nowadays we have 290 local authorities and almost every municipality has their own control network and in some areas, there is more than one network because a forming of two or more municipalities into one has taken place.

In Sweden, the responsibility for geodetic control networks is divided between the local authorities and Lantmäteriet (National Land Survey of Sweden). The main cause for this is different aims of the systems. The responsibility for Lantmäteriet is to establish ground control for official mapping in small scales. The local authorities establish control networks for urban development.

Lantmäteriet is the national geodetic authority but has no power against municipalities and other authorities. Lantmäteriet cannot do anything else than give proposals and advice to the local authorities concerning their reference systems.

Lantmäteriet is responsible for all national geodetic networks. The local authorities are responsible for their own networks. In 2003, a decision was made to replace the national reference frame RT 90, which is based on the Bessel 1841 ellipsoid, with a globally aligned reference frame, SWEREF 99, which will be appropriate for a long time. Preparations for this replacement now have started, both locally and nationally. Lantmäteriet will officially change in January 2007. This means e.g. that all data stored today in RT 90 in our databases should be transferred to SWEREF 99 and all our official products as maps should be in SWEREF 99. Several other governmental agencies will follow shortly after.

2.2 Work done on National and Local Level

The reference system used nationally must meet several criteria. It must be modern in such a way that positioning using modern technologies should be possible without destroying the high accuracy that modern instruments can achieve. It should make it possible to easily and efficiently exchange data with neighbouring countries and other users within the country, which means that the connections to other reference frames must be well known or we should work in the same reference frame.

The implementation of SWEREF 99 as the national reference frame for GNSS was done during 2001. This means that positioning in relation to the SWEPOS™ stations is done in an accurate reference frame well connected to our neighbouring countries and a major part of Europe. This is not the case with RT 90.

Locally, we nowadays have several hundreds of different geodetic networks. Lantmäteriet recommends the local authorities to tie their local networks to the national reference frame or – preferably – to use the national reference frame. To help the users today, we have a project running called RIX 95 that will calculate transformation parameters between SWEREF 99 and the local networks, meaning that it locally will be easier to implement and utilize GNSS. One of the results from the development of the transformation parameters is a map of the residuals that exist and these residuals are used when computing the correction model.

2.3 Choice of Method for Rectifying Distorted Co-ordinate Systems

To reduce the local deformations of the local co-ordinate systems, studies of several rectifying methods were carried out (Svanholm, 2000; Alfredsson, 2002). Four methods were tested and those were point-by-point transformation based on both Helmert transformation and Affine transformation, interpolation of residuals in triangles based on the common control points themselves and with fictitious common control points.

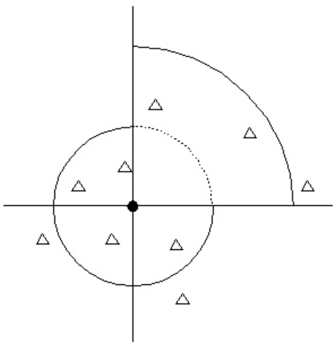

Figure 1: The point-by-point transformation search method.

The point-by-point transformation method searches for common control points within a radius around the point that is to be rectified. Some criteria must be fulfilled, like the number of common control points and there must be common control points in all directions around the point. If these criteria are not fulfilled in the first iteration the radius is increased until all criteria are achieved. To transform the point, a Helmert or Affine transformation based on the common control points found is performed.

Methods with interpolation in triangles are based on a Delaunay triangulation between the common control points. The interpolation[1] is then carried out inside the triangles based on the residuals of each common control point. The residuals are the result from the calculation of transformation parameters (see section 3.1).

When the interpolation is based on the common control points themselves there is no smoothing of the residuals, e.g. a measuring error is not filtered. This smoothing can be performed by first producing a set of fictitious common control points and then do a Delaunay triangulation between them. To produce a set of fictitious common control points a type of point-by-point transformation was carried out based on a Helmert transformation.

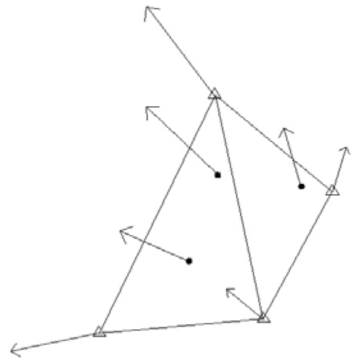

Figure 2: Interpolation of residuals.

The studies showed that the point-by-point transformation methods were smoothing too much and there were jumps in the rectifying results. Therefore both the point-by-point transformation methods and the interpolation of residuals based on fictitious common control points did not give as good result as the interpolation of residuals based on the common control points themselves.

Based on the results from the studies, we decided to use the method with interpolation of residuals in Delaunay triangles as our rubber sheeting algorithm.

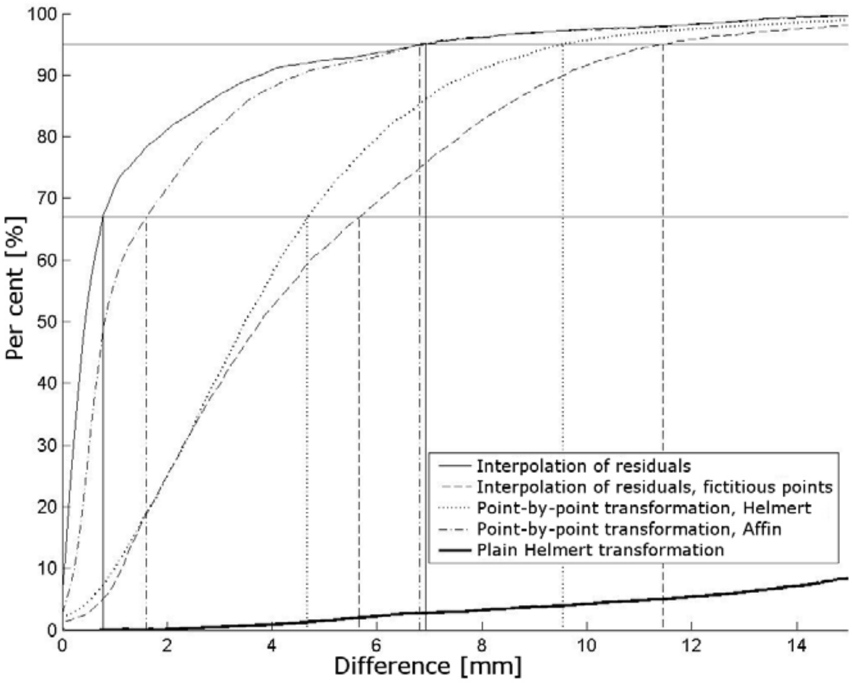

Figure 3: Diagram that shows the distribution of the differences. The vertical and horizontal lines show the difference at 67 % and 95 %.

[1] The interpolation is an Affine transformation based on the three control points forming the triangle.

3. CORRECTION MODELS

The correction models are based, as earlier mentioned, on the residuals existing after the transformation from SWEREF 99 to the local co-ordinate system. The transformation is based on the parameters from the RIX 95 project. The rectification of the local reference frame is then done by a rubber sheeting algorithm (see section 2.3 and 3.4 for further information) to obtain a homogeneous system.

3.1 RIX 95 Transformation Parameters

Since 1995, a project called RIX 95, involving GPS measurements on triangulation stations in the national network and selected control points in the local networks has been in operation. The work is financed by a group of national agencies. The principal aims are to connect local reference frames to the national reference frames (SWEREF 99 and RT 90) and to establish new control points easily accessible for local GNSS measurements.

Concerning the connection of local reference frames, transformation parameters based on different transformation models are developed. The parameters are mainly based only on *direct projection* (Engberg & Lilje 2006) with Transverse Mercator, but in some cases also combined with similarity transformation in two or three dimensions. Now (June 2006) transformation parameters for 220 of the 290 Swedish municipalities are available.

3.2 Selection of Control Points

Common control points should be selected in the areas where a correction is needed, i.e. where there is a local control network or if the area is covered by geographical databases. As the rubber sheeting algorithm used is a triangular model (see section 2.3 and 3.4) it is important that the control points surround the community or local control network. Otherwise the control points in one community can affect the correction values in another community.

In the smallest communities or local control network parts it is recommended that at least 3-6 common control points are selected for use in the correction model. For larger control networks a densification of the common control points of course is needed. We recommend a control point spacing of 500-700 metres, but from the beginning a more sparse distribution of control points can be used. The deformation analysis (see section 3.3) will show if a densification of the control point distribution is needed in some areas. The objective is to map the deformations of the local control networks.

Lantmäteriet recommends mainly two methods for surveying of these control points, namely rapid static GNSS survey or RTK survey (Kempe, 2006). The co-ordinates of the control points must be determined in a controlled way, as the future control network accuracy is directly dependent of these control points. The errors to be modeled by the correction model shall concern system errors only and not errors from the survey.

3.3 Evaluation of Local Control Network Deformations

The GNSS-surveyed common control point co-ordinates are transformed to the local co-ordinate system using the RIX 95 transformation parameters (see section 3.1). The residuals from the comparison of these co-ordinated to the “true” local co-ordinates are then analysed.

To facilitate the analysis of the variations of the deformations of the local control network, a Matlab tool has been developed.

- One of the two residual vectors along a triangle side of the Delaunay triangulation is moved to the other one.
- A vector subtraction is done to get a resulting vector of length *d*.
- The *d* value is then handled as a scalar and is moved to the middle of the triangle side. The *d* values are normally weighted inversely to the square of the point distance or absolute, i.e. no weighting. The first case is most often used, but the latter case is a useful complement in some cases.
- Perform the vector subtraction and transfer of the *d* values for all triangle sides.
- The *d* values can then be plotted as an image map (for an example, see figure 6) using e.g. Matlab.

Figure 4 below will illustrate how the analysis tool works.

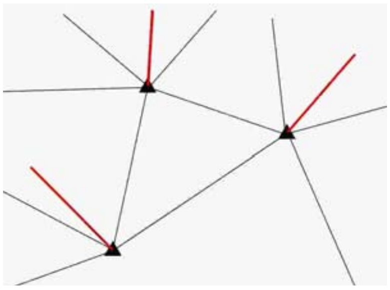

Figure 4a: One triangle of a Delaunay triangulation serves as an example of the Matlab analysis tool.

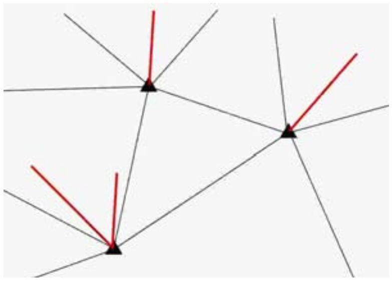

Figure 4b: The residual from one control point is then moved along the triangle side to do a vector subtraction.

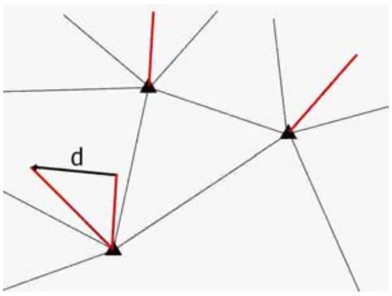

Figure 4c: The length of the resulting vector d of the vector subtraction is measured.

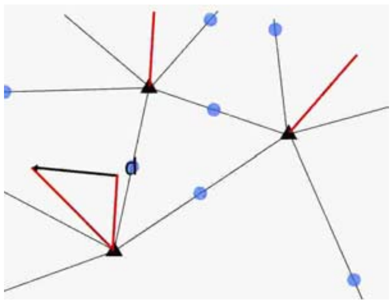

Figure 4d: The d value is moved to the middle point of the triangle side. The same procedure is carried out for all triangle sides of the triangulation. Figure 6 shows a Matlab plot of these d values.

An example can further illustrate the procedure of analysing the deformations of the local control network. Figure 5 shows the common control point residuals of the Swedish municipality Tranås after transformation with RIX 95 transformation parameters. There are 24 control points and the size of the largest vector is a little more than 10 centimetres.

Figure 5: The residuals after transformation of control points with RIX 95 transformation parameters.

The Matlab plot of the Tranås case is shown in figure 6 below. The red (dark) colour shows areas with large variations in the deformations where there might be a need to have a further look into the deformations, whereas the bright colour are areas of higher homogeneity.

The next step in the Tranås case was to further map the variations and thus another six common control points were included (see figure 7). At the same time the two deviant control points in figure 6 were excluded, as they did not represent the local control network very well.

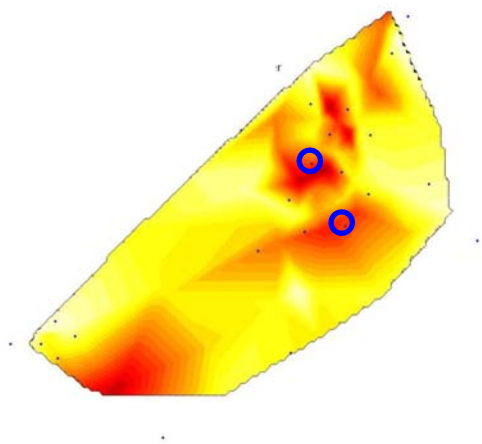

Figure 6: The variations of deformations of the local control network of Tranås, based on the residuals shown in figure 5. The deformations are weighted inversely to the square of the point distance. The two marked common control points were deviant, and finally excluded.

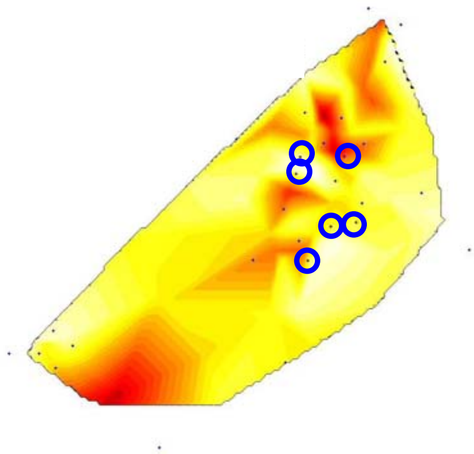

Figure 7: The variations of deformations of the local control network of Tranås, after inclusion of six supplementary common control points and exclusion of the two deviant points shown in figure 6.

The selection of common control points and analysis of the variations of the deformations is an iterative process that continues until the accuracy is adequate for rectifying of the local control network.

3.4 Delivery and Use of Correction Models

A default correction model delivered by Lantmäteriet is created using our own software Gtrans. It is based on Delaunay triangles and a rubber sheeting algorithm (interpolation of residuals; see section 2.3) within each triangle. These models can be used with e.g. Lantmäteriet's software packages Gtrans, AutoKa-PC and ArcCadastre.

Other software packages, which cannot make use of the Gtrans correction model, might utilize other rubber sheeting algorithms. The data that serves as a basis for the Gtrans correction model can be delivered to the users of such softwares, for implementation of their own rubber sheeting algorithms.

The correction model is used as figure 8 shows, in combination with the RIX 95 transformation parameters.

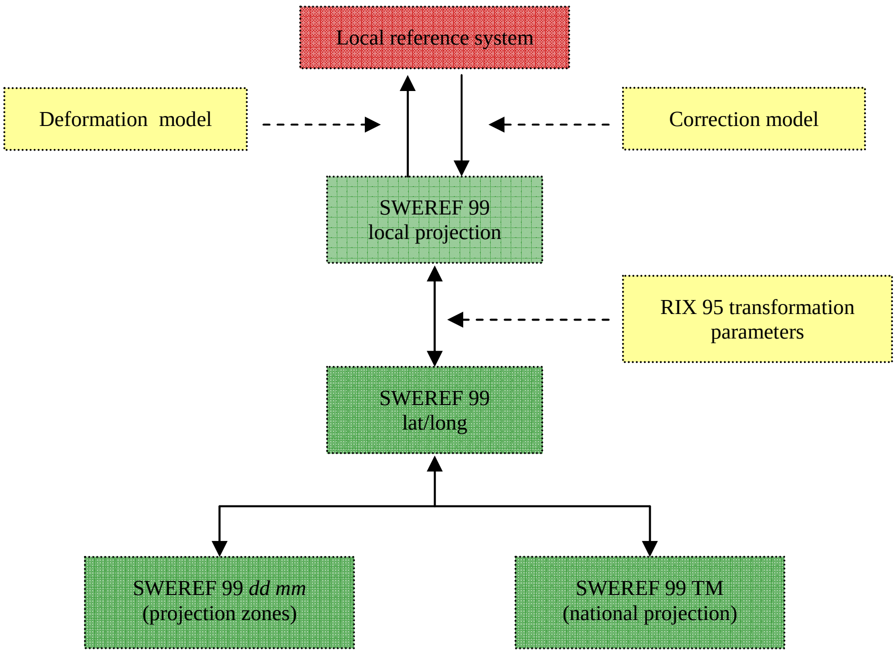

Figure 8: Transformation schedule for local reference systems to and from SWEREF 99.

The input data to the correction model can also be used to create a deformation model, normally in grid format. The deformation can then be used in GNSS receivers, to distort the geometry of the GNSS surveying to fit the local co-ordinate system. This way of using the deformation model can be seen as an intermediate step, before the step is taken to change the local reference system to a SWEREF 99-based system. Until the change is done, the new surveys can be distorted and when it is time to change, they can be transferred to SWEREF 99 in the same way as the old data.

4. CONCLUDING REMARKS

When looking at different functions for interpolation of these corrections one has to take into account the nature of the original residuals – what deformation has caused them? Normally the control network is composed of parts originated from various periods and established with varying techniques. Within those parts you can expect deformations that are continuous but with discontinuities between them. Obviously, the interpolation function should reflect these conditions. Other types of interpolation techniques that have been discussed are 'inverse distance to a power', kriging, and 'natural neighbour' but only the last-mentioned is a possible choice if we are going to test a new function.

Our experiences so far are that the approach with simple interpolation in Delaunay triangles has turned out to produce correction models good enough for their purposes. As an example, the correction model covering five municipalities (600 km^2) in southern Sweden gives results better than 2 cm. That is the same level of accuracy as both the RTK measurements and the position in the old network.

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Ms Kempe graduated in 2000 from the University College of Gävle as a Mapping and Surveying Engineer (BSc.). She is since then working at the Geodetic Research Department of Lantmäteriet.

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Mr Alfredsson is a member of the Swedish Map and Measuring Technique Society.

Lars E Engberg
Mr Engberg obtained his Master's degree from the Royal Institute of Technology in Stockholm 1973. He has been working as a lecturer in Geodesy at the School of Surveying for many years. Between 1989 and 1996 he was at the City Surveying Department in Stockholm and responsible for the establishment of an improved reference network in Greater Stockholm. Since 1996 he is working at the Geodetic Research Department at Lantmäteriet. At present, he is involved in a national project aiming to implement the new reference frame SWEREF 99 as a national standard. He is also engaged as an international adviser.
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Mikael Lilje
Mr Lilje graduated in 1993 from the Royal Institute of Technology as a Land Surveyor with emphasis on Geodesy and Photogrammetry. He is working at Lantmäteriet since 1994 with various topics, mainly at the Geodetic Research Department. Currently he is the head of a group working with reference frame and co-ordinate system questions.
Mr Lilje is chair of the Swedish Map and Measuring Technique Society and chair of the FIG Commission 5 Working Group on ”Reference Frame in Practice”. Mr Lilje was also secretary of FIG Commission 5 during the period 1998 – 2002.

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