Splines and Kriging - the use of two methods for shell structures shape analysis

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Key words: shell structures, splines, kriging

SUMMARY

The study concerns the subject of periodic shape control of the shell objects surface. These objects due to the nonlinear shape of the shell require the use of appropriate methods of approximation, enabling creation of models correctly reflecting not only the theoretical shape of the shell, but also its local deformation, resulting in the process of construction and maintenance of buildings. This task can be realized on the basis of two solutions that have established position among the methods of approximation i.e.: spline functions and the method of kriging. Each of them has its particular features which the authors tried to highlight by running the appropriate tests on the selected engineering structure. These characteristics determine the quality of approximations; especially in case of not enough regular set of observations of the object. The paper is an attempt to answer the question, what level of accuracy can be expected from the approximations by both methods in case of observations carried out regularly, and in case of local deterioration in their regularity. Analysis of errors of each method; enabled authors to propose combination of their features. It tended to obtain more robust models to errors resulting from local deterioration in regularity of observational data.

STRESZCZENIE

Niniejsze opracowanie związane jest z okresową kontrolą kształtu powierzchni powłokowych obiektów inżynierskich. Obiekty te z uwagi na nieliniowy kształt płaszcza wymagają użycia odpowiednich metod aproksymacyjnych, pozwalających na utworzenie modeli właściwie opisujących nie tylko teoretyczny kształt powłoki, ale też jej lokalne deformacje, powstałe w trakcie procesu budowy i późniejszej eksploatacji budowli. Zadanie takie można zrealizować w oparciu o dwa rozwiązania mające ugruntowaną pozycję wśród metod aproksymacji: funkcje sklejane oraz metodę krigingu. Każda z tych metod posiada swoje charakterystyczne cechy, które autorzy starali się uwypuklić poprzez wykonanie odpowiednich testów na przykładzie wybranego obiektu inżynierskiego. Cechy te decydują o jakości przybliżeń, szczególnie w wypadku dostarczenia mało regularnego zbioru obserwacji obiektu. Opracowanie stanowi próbę odpowiedzi na pytanie, jakich dokładności przybliżeń można oczekiwać od obydwu metod w przypadku obserwacji prowadzonych regularnie, oraz w wypadku lokalnego pogorszenia ich regularności. Analiza błędów każdej z metod, pozwoliła autorom w końcowej części opracowania na zaproponowanie połączenia ich cech. Zmierzało ono do otrzymania modeli bardziej odpornych na błędy wynikłe z lokalnych zaburzeń równomierności rozkładu danych obserwacyjnych.

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1. INTRODUCTION

The assessment of the geometrical state of building structures is one of the important tasks within the scope of surveying engineering. Nowadays, the shape of the structures is not only dictated by economics or functionality but also by the architectural vision of the designer. The particular types of objects are the shell structures with their nonlinear shape which requires appropriate computational methods enabling analysis of deformation. It is a very important task from the viewpoint of functionality and sometimes even from the statics of objects of that kind. Depending on the building function it is necessary to detect the deformation on the level of several or tens of millimeters. This kind of deformation is the most often determined in reference to the designing state, as-built survey or recently carried out, therefore during the periodical control of the object's shape. The crucial stage in this kind of analysis is always providing the suitable set of observation of the shell structure what is carried out in a gridded way usually by means of reflector-less polar methods. More demanding task is, however, the choice of appropriate method of changing gridded set of observations into precise, continuous model of the surface, which will be used for shape analysis later. Mathematical tools applied to this problem ought to be possibly general for different types of objects as well as methods of their observations. They should assure the reliable comparison of actual survey results with the given ones, previous state of structure. Such comparison must be possible at any chosen point of the model. The opportunity of appropriate presentation of all local deformation of the surface is crucial in this respect. It entails the necessity of choice of computation tools, enabling obtaining good results of approximation, independently on the type of function which was the base for evaluating the shape of observed surface.

Such task can be realized on the basis of two solutions having grounded position among methods of approximation. The first of them is spline interpolation enabling creation of smooth models with continuous curvature, well describing the shape of shell objects. Spline functions give also the opportunity of reflecting local, nonlinear deformation of the object. The second method of interpolation used in the research is kriging. The term kriging stands for a variety of spatial prediction methods linear and nonlinear as well as multivariate ones. It is a statistical method of interpolation (exact interpolator) but also can be presented in smoothing form (filtering), strongly connected with the theory of random fields. Kriging predicts the unknown values of the quantity from the observed ones with known spatial (also temporal) locations. The base for the method of kriging is a function reflecting spatial (or temporal) structure of the data (semivariogram or covariance function) i.e. spatial autocorrelation, continuity and variability. As far as authors' knowledge goes kriging was usually used in earth sciences which are burdened with some load (sometimes quite considerable) of uncertainty, hence this study in the field of surveying engineering may seem to be new.

The goal of the paper is the comparison of interpolation quality of the two aforementioned methods on the basis of exemplary engineering shell structure. Both methods were already compared (Dubrule 1983, Laslett 1994, Watson 1984, Boer Beurs Hartkamp 2001), but rather in abstract terms. This presentation covers practical problem concerning approximation of the shape of shell structures, which is of great importance in the field of surveying engineering. The intention of authors is to attract attention to some characteristic errors which can be introduced in object's shape description by the two methods what leads to deformation of the model. However, the two methods can be complementary giving the results far better then each of them separately. In this paper, authors indicated one of such exemplary solution, which is now under detailed research.

2. THE PRINCIPLES OF OBJECT MODELING BY SPLINE FUNCTIONS

The essentials of spline functions are included in a number of papers (Ahlberg, Nilson, Walsh 1967, De Boor 1978). In this paper, the most common form of splines, broadly used in CAD software, B – splines have been presented.

The splines are compositions of polynomials of low, usually 3[rd] degree, joined in knots with preserving the continuity of curvature. Due to the numerous favorable properties (Boehm Paluszny Prautzsch 2002, Kiciak 2000), their parameterized notation is more often used in place of the open notation. The selection of t_i knots takes place in the parameterization process, related to mutual distances between the points being interpolated. The most common form of splines are the B-splines (Boehm Paluszny Prautzsch 2002, Bojanov Hakopian Sahakian 1993, Diercx 1995), determined as linear combination of base polynomials N_i^m with coefficients d_i :

$$S_i(t) = \sum_{i=0}^{n-m-1} d_i N_i^m(t) \qquad t_i = \{t_0,..,t_n\} \tag{1}$$

The polynomials of *m*-degree are described by the recursive formula of Mansfield-de Boor-Cox:

$$N_i^0(t) = \begin{cases} 1 \quad dla \quad t \in [t_i, t_{i+1}) \\ 0 \quad dla \quad t \notin [t_i, t_{i+1}) \end{cases}$$

$$N_i^k(t) = \frac{t - t_i}{t_{i+k} - t_i} N_i^{k-1}(t) + \frac{t_{i+k+1} - t}{t_{i+k+1} - t_{i+1}} N_{i+1}^{k-1}(t) \tag{2}$$

The equation of B-spline of 3[rd] degree, on the interval $t \in [t_i, t_{i+1})$ is given by:

$$S_{i-3}(t) = d_{i-3}N_{i-3}^3(t) + d_{i-2}N_{i-2}^3(t) + d_{i-1}N_{i-1}^3(t) + d_i N_i^3(t) \tag{3}$$

The function interpolating the set of points p_i is determined by the system of equations:

$$\begin{bmatrix} a_1 & a_2 & a_3 & . & . & 0 \\ N_0^3(t_3) & N_1^3(t_3) & N_2^3(t_3) & 0 & . & 0 \\ 0 & N_1^3(t_4) & N_2^3(t_4) & N_3^3(t_4) & . & 0 \\ . & . & . & . & . & . \\ 0 & . & . & N_{n-m-3}^3(t_{n-m}) & N_{n-m-2}^3(t_{n-m}) & N_{n-m-1}^3(t_{n-m}) \\ 0 & . & . & b_1 & b_2 & b_3 \end{bmatrix} . * \begin{bmatrix} d_0 \\ d_1 \\ . \\ . \\ d_{n-m-2} \\ d_{n-m-1} \end{bmatrix} = \begin{bmatrix} 0 \\ p_0 \\ . \\ . \\ p_{n-m-3} \\ 0 \end{bmatrix} \quad (4)$$

where: a_i , b_i - terms resulting from the boundary conditions applied, in place of p_i the x_i , y_i coordinates of those points should be substituted.

The methods for curve formation discussed are also applicable to surface construction. Currently, the method of *lofting* is in common use for surface construction, allowing good approximation of the structures being measured. It relies on spanning the surface over curves (1), previously formed based on the measuring points. The surface is, therefore, formed in two stages and can be described by tensor product $S = F \otimes V$ of the spaces of the basis function of the type (2): $N_i^3(t) \in F$ and $N_j^3(u) \in V$:

$$S_{i,j}(t,u) = \sum_{i=0}^{n-4}\sum_{j=0}^{r-4} d_{i,j} \cdot N_i^3(t) \cdot N_j^3(u) \qquad t_i = \{t_0,..,t_n\} \qquad u_i = \{u_0,..,u_r\} \qquad (5)$$

In the first step, *lofting* creates curves $S_i(t)$ on the basis of interpolation points, then, in next step the procedure is analogical with exception that previously created interpolation curves become now the “interpolation points” and the “curve” being actually created is the surface $S_{i,j}(t,u)$. The surfaces may also occur in NURBS form.

During the process of spline functions forming (both curves and surfaces), the factors like: type of parameterization (choice of knots) applied, boundary conditions and the spatial distribution of points have significant meaning.

In order to obtain good results in approximation, the parameterization must be related to mutual distances between measured points (Boehm Paluszny Prautzsch 2002, Lenda 2005). For the objects characterized by gentle changes of curvature, the selection of nodes should be proportional to the distances between measured points and for the objects with fast changing curvature the appropriate parameterization will be the square root of distances between points. Both types of parameterization can be applied alternately to the same function depending on its complexity.

The choice of boundary conditions (De Boor 1978, Kiciak 2000) has the influence over extreme parts of spline function (three initial and three ending ones) and because of its subjectivity may lead to deformations on function’s ends. Thus, these parts of spline function should be treated with limited trust. However, when sampling is regular and appropriate type of parameterization applied, the influence of boundary conditions becomes less significant.

The most important thing, however, is the sampling. In case of irregular sampling, the “wavelets of the function” are immediately noticeable this can be eliminated to some degree by applying appropriate parameterization (selection of knots). In general, the more complex is the shape of the object the more regular sampling has to be provided. Since, during the survey of the object, the desired regularity of sampling cannot be always achieved, thus its lack can be the main contribution to the errors of the model. The paper focuses on the situation mentioned above and presents the idea that gives the possibility of increasing the accuracy of the approximation by the use of the second method discussed – the kriging method.

3. THE PRINCIPLES OF KRIGING METHOD

Kriging is a statistical method of spatial or temporal prediction (also estimation, interpolation, smoothing depending on the purpose), which assumes that the observed data z(**s**) is a partial realisation of the Z(**s**) random function. While formulating a mathematical model of the phenomenon represented by certain random field Z(**s**) (random function), general decomposition into two parts is often involved: overall trend (drift) of the data and fluctuation.

$$\mathbf{Z}(\mathbf{s}) = \boldsymbol{\mu}(\mathbf{s}) + \boldsymbol{\varepsilon}(\mathbf{s}), \quad \mathbf{s} \in D \subset R^n \tag{6}$$

where:
$\boldsymbol{\mu}(\mathbf{s})$ – trend (drift)
$\boldsymbol{\varepsilon}(\mathbf{s})$ – fluctuation (structured error term)
s – spatial location (in case of R^2 [x,y] coordinates)

The drift, being a global model of low frequency (depending on the assumptions it can be constant and known mean, constant but unknown mean or location - dependent mean), describes the average performance of the phenomenon in the spatial domain, whereas the fluctuation, being a high frequency component, reflects discrepancies of the phenomenon around its average value – drift.

Kriging is often referred to as BLUP (Best Linear Unbiased Predictor), which means that it minimizes the mean square error of prediction (8) (best), is a linear combination of observed data (linear), the average error of the predictor is equal to zero (7) (unbiased).

$$E[p(\mathbf{Z}, \mathbf{s}_o) - Z(\mathbf{s}_o)] = 0 \tag{7}$$

$$E[p(\mathbf{Z}, \mathbf{s}_o) - Z(\mathbf{s}_o)]^2 \rightarrow \min \tag{8}$$

where:
$p(\mathbf{Z},\mathbf{s}_o)$ – predictor of the unobserved (unknown) value Z at $\mathbf{s}_o$
E – expected value operator

To enable construction of optimal predictors in the minimum mean square sense we need to introduce some function reflecting the structure of the data i.e. semivariogram or covariance function. Only practical information on semivariogram will be given in order to avoid unnecessary theoretical consideration. Semivariogram is a structure function which reflects spatial continuity and variability of the random function (considered phenomenon). Inference as to the semivariogram shape is based on an empirical semivariogram, estimated from the data and a priori knowledge of the phenomenon (continuity and regularity of the phenomenon, potential measuring errors etc.). Figure 1 depicts basic parameters of the semivariogram: range of influence "a" (radius of auto-correlation), "nugget effect" "c_o", partial "c" and complete sill "c_o+c_1". The range of influence corresponds to the distance between observations, for which the spatial correlation is visible. The "nugget effect" is a product of measuring errors, lack of information on the phenomenon behaviour at distances shorter than the minimum distance between observations in the studied *D* domain, and rapid changes of the phenomenon itself. The total sill is the sum of "nugget effect" and the partial sill ($c_o + c_1$), in the Figure 1 shown as a flattening, which the semivariogram reaches beyond the range of influence.

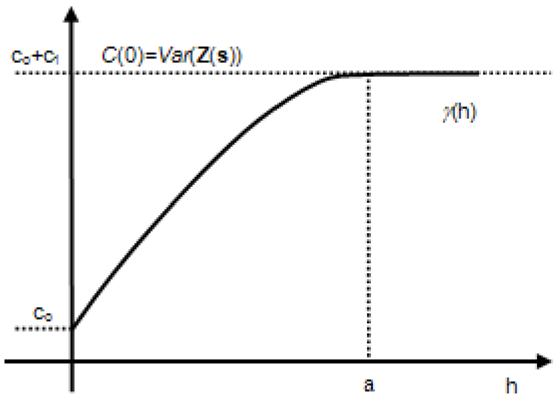


Fig. 1 Semivariogram function and its characteristic parameters

The most important issue while modelling the semivariogram, is its behaviour near the origin and optimum fit in several initial distances, for which the spatial correlation is apparent, as the observations located within the range of influence will be of utmost importance for predicting the non-observed value. The semivariogram, while approaching the origin of the system of coordinates, may be of parabolic shape (square), which describes highly continuous and regular process, and also can indicate the existence of a strong drift; linear shape, which reflects continuous but less regular process than the parabolic one; the discontinuity in the origin of the system of coordinates indicates an irregular process at short distances (Armstrong 1998, Journel and Huijbregts 2003).

The nature of the issues under consideration forces the use of such kriging method, which takes under account a large-scale behavior of the objects, which are often constructed on the basis of more or less complex mathematical functions. This kind of method is universal kriging (kriging with a trend/drift model). Universal kriging predictor of unknown quantity is expressed as linear combination of observed data with coefficients $\boldsymbol{\lambda}$:

$$\hat{Z}(\mathbf{s}_o) = p(\mathbf{s}_o, \mathbf{Z}(\mathbf{s})) = \boldsymbol{\lambda}^T \mathbf{Z}(\mathbf{s}) \qquad (9)$$

where:
$\boldsymbol{\lambda}$ – vector of weight coefficients (kriging weights)
$\mathbf{Z}(\mathbf{s})$ – observed data,

To derive the formula for universal kriging predictor it is assumed that both for the observed (10) and non-observed (11) data, the generalized linear models hold:

$$\mathbf{Z}(\mathbf{s}) = \mathbf{F}(\mathbf{s})\boldsymbol{\beta} + \boldsymbol{\varepsilon}(\mathbf{s}) \qquad (10)$$

$$Z(\mathbf{s}_o) = \mathbf{f}(\mathbf{s}_o)^T \boldsymbol{\beta} + \varepsilon(\mathbf{s}_o) \qquad (11)$$

where $\mathbf{F}(\mathbf{s})$ and $\mathbf{f}(\mathbf{s}_o)$, are respectively, the matrix and vector of know basic functions in a form of monomials, e.g. 1, x, y, x^2,y^2,*xy*,... and $\boldsymbol{\beta}$ vector of unknown trend coefficients.
To assure the predictor (9) to be unbiased we obtain the so called "*universality conditions*":

$$E\left[Z(\mathbf{s}_o) - \hat{Z}(\mathbf{s}_o)\right] = E\left[Z(\mathbf{s}_o) - \boldsymbol{\lambda}^T \mathbf{Z}(\mathbf{s})\right] = 0 \Rightarrow \boldsymbol{\lambda}^T \mathbf{F}(\mathbf{s}) = \mathbf{f}(\mathbf{s}_o)^T \qquad (12)$$

The mean square error of prediction is given by:

$$E\left[Z(\mathbf{s}_o)-\hat{Z}(\mathbf{s}_o)\right]^2 = -\boldsymbol{\lambda}^T\boldsymbol{\Gamma}\boldsymbol{\lambda}+2\boldsymbol{\lambda}^T\boldsymbol{\gamma} \tag{13}$$

where:

$\boldsymbol{\Gamma}$ – matrix of semivariances (between observed data)

$\boldsymbol{\gamma}$ – vector of semivariances (between observed data and unobserved to be predicted)

Hence, by combining (12) with (13) the objective function to be minimized with the use of Lagrange multiplier can be expressed as:

$$\Psi(\boldsymbol{\lambda},\boldsymbol{\kappa}) = E\left[Z(\mathbf{s}_o)-\hat{Z}(\mathbf{s}_o)\right]^2 - 2\boldsymbol{\kappa}^T\left[\mathbf{F}(\mathbf{s})^T\boldsymbol{\lambda}-\mathbf{f}(\mathbf{s}_o)\right] = -\boldsymbol{\lambda}^T\boldsymbol{\Gamma}\boldsymbol{\lambda}+2\boldsymbol{\lambda}^T\boldsymbol{\gamma}-2\boldsymbol{\kappa}^T\left[\mathbf{F}(\mathbf{s})^T\boldsymbol{\lambda}-\mathbf{f}(\mathbf{s}_o)\right] \tag{14}$$

Taking partial derivatives with respect to the coefficients $\boldsymbol{\lambda}$ and the Lagrange multipliers $\boldsymbol{\kappa}$ and setting them to zero we obtain the universal kriging system of equation of the form:

$$\begin{cases} \dfrac{\partial\Psi(\boldsymbol{\lambda},\boldsymbol{\kappa})}{\partial\boldsymbol{\lambda}} = -\boldsymbol{\Gamma}\boldsymbol{\lambda}+\boldsymbol{\gamma}-\mathbf{F}(\mathbf{s})\boldsymbol{\kappa} = 0 \\ \dfrac{\partial\Psi(\boldsymbol{\lambda},\boldsymbol{\kappa})}{\partial\boldsymbol{\kappa}} = -\mathbf{F}(\mathbf{s})^T\boldsymbol{\lambda}+\mathbf{f}(\mathbf{s}_o) = 0 \end{cases} \tag{15}$$

or in matrix form:

$$\begin{bmatrix} \boldsymbol{\Gamma} & \mathbf{F} \\ \mathbf{F}^T & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\kappa} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\gamma} \\ \mathbf{f} \end{bmatrix} \Rightarrow \boldsymbol{\Gamma}_U \cdot \boldsymbol{\lambda}_U = \boldsymbol{\gamma}_U \tag{16}$$

The solution to the above system of equations is the $\boldsymbol{\lambda}_\mathbf{U}$ vector comprising a set of optimum coefficients $\boldsymbol{\lambda}$ (kriging weights) and Lagrange multipliers $\boldsymbol{\kappa}$, which, after substitution to (9) and (13) result in estimating an unknown value $Z(\mathbf{s}_o)$ and prediction variance (kriging variance).

4. COMPARISON OF RESULTS OBTAINED BY SPLINE FUNCTIONS AND KRIGING

Comparison of the quality of the interpolation using the above-described methods was carried out on the example of regularly observed shell object, having only local sampling irregularities. It is a fragment of the outer part of the self-supporting dome being the property of the RMF radio in Nieporaż, Poland (Fig. 2). The tests relied on the accuracy check of reproducing the position of points with known coordinates basing on the generated model. The authors wanted to separate the error component introduced by approximation method from the remaining uncertainties due to measurement errors and those related to object realization. For this purpose, after the measurement, the quadrics (ellipsoid) was fitted to the set of observed points, on which the measured points were projected. In this way, the points lying on an ideal surface were obtained, but preserving the sampling as it was during the measurement. Since, in the two methods the final results are mainly biased by the level of regularity of the sampling, this factor was isolated as crucial for the quality of approximation. Afterwards, the grid of test points was generated and was used for testing the two methods. Thus, the projected (observed) as well as tested points (Fig. 3) were situated exactly on the mathematical surface of the ellipsoid. In this case, any uncertainties present during the reproducing process of test points will be the result of the applied method. For model construction there were 190 points used and for comparison purposes 240 test points.

On the basis of the given test material, the discussion of results was provided. To generating and analyzing the quality of spline surfaces Rhinoceros 4.0. software was used and for kriging method Surfer v. 8 and authors' own software.

Both, splines and kriging, may by applied in interpolating or approximating form. For comparison, to be clear and meaningful, all tested surfaces in this study, were created using interpolation.

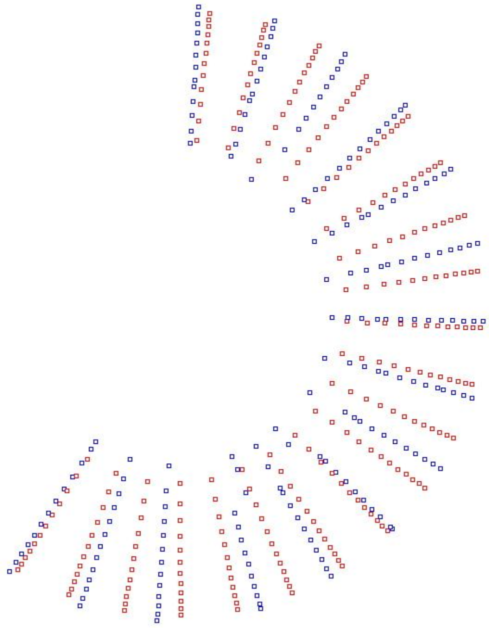


Fig. 2. The dome of RMF Radio in Nieporaż, Poland

Fig. 3. Points given (blue) and points to be reproduced (red), top view

4.1. Spline interpolation

According to the assumptions of *lofting* method, basing on given points the curves had been constructed (Fig. 4), on which the surface was spanned later. In both cases, the parameterization proportional to the distance between measured points was used. Figure 4 depicts places with irregular observations, which may introduce potential deformations of the surface. Then, the deviations of test points from the generated surface were determined, which are really the errors of considered method. The quantitative results that inform about the number of points with deviations within given accuracy intervals have been listed in Table 1. The qualitative and quantitative results depicting the spatial distribution of deviations within given intervals are shown in Fig 5.

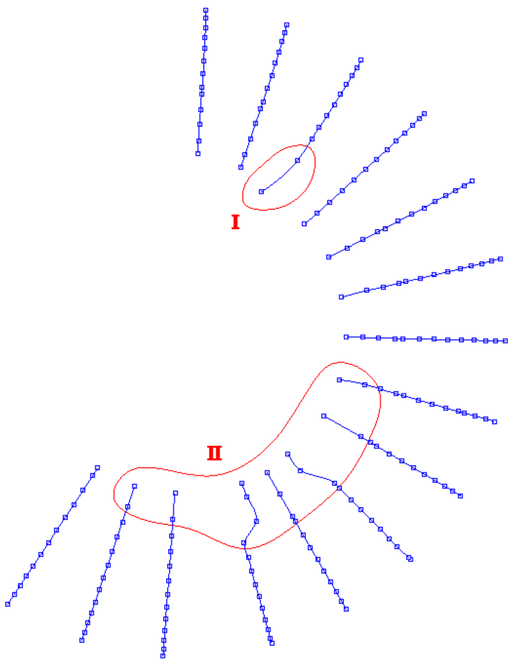
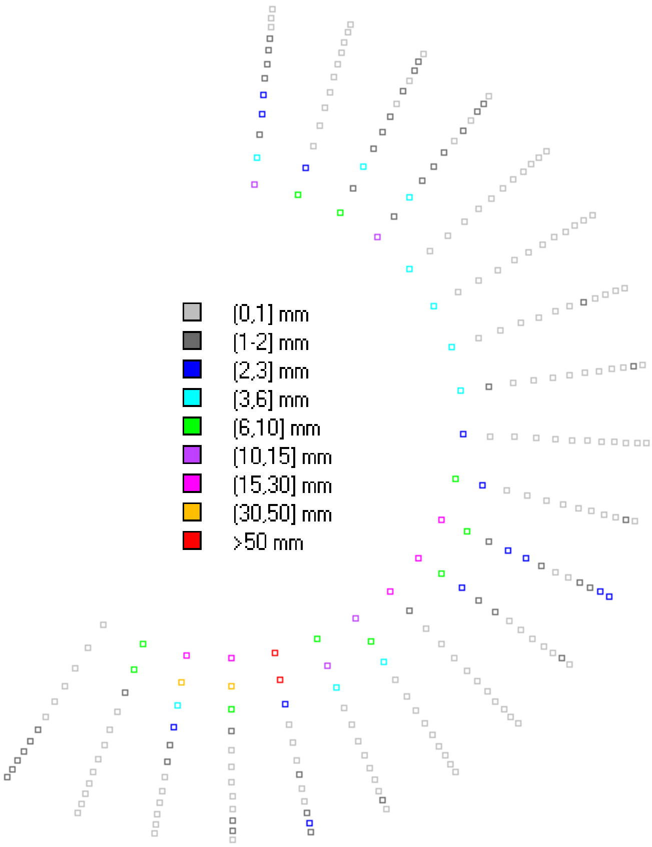


Fig. 4. Spline curves created on the basis of given points. Envelope marks areas with the greatest distortion of sampling.



interval of deviations [mm]	number of points	contribution %
(0,1]	148	61.7
(1,2]	46	19.2
(2,3]	13	5.4
(3,4]	4	1.7
(4,5]	3	1.3
(5,6]	3	1.3
(6,7]	2	0.8
(7,8]	3	1.3
(8,9]	3	1.3
(9,10]	2	0.8
(10,12]	1	0.4
(12,14]	2	0.8
(14,16]	1	0.4
(16,18]	0	0.0
(18,20]	0	0.0
(20,25]	2	0.8
(25,30]	3	1.3
(30,35]	1	0.4
(35,40]	1	0.4
(40,60]	1	0.4
(60,80]	1	0.4

Fig 5. Spatial distribution of deviations obtained by spline interpolation

Tab. 1. The magnitude of deviations obtained by spline interpolation

It is worth starting the assessment of the quality of approximation from places where observations were carried out regularly. In all these places, the accuracy obtained was at a very good level. Approximately 75% of deviations got the value less then 1 mm, 25% fell into the interval (1, 2] mm. Only a few points on the edge of the surface got the deviations of about 3 mm, which may result from the influence of the boundary conditions. Thus, the accuracy of approximations is better then the real assumed measurement errors (Lenda 2003), characteristic for reflector-less polar methods (over a dozen mm) commonly used in measuring such objects.

The deviations observed within the region I (several to over a dozen of mm) occurred as a result of local, too sparse measurement points. Similar situation is present on the left and right edge of the region of irregular spatial distribution denoted as II. In this case, however, the deviations rapidly change their values (15, 30] mm, which is due to the proximity of large irregularities in the spatial distribution, located in the center of the area II. In its direct proximity, deviations in some places are close to the value of 50 mm, and in the center of the area nearly 80 mm.

Attention ought to be paid to the entire upper strip of the surface where the deviations are greater than for other edges. Generally, it shows less regularity of the measurements than other edge strips what with the connection to the influence of boundary conditions and distortion of distribution of points in surrounding areas contributes to the lower quality of interpolation.

In conclusion, besides the places of visible distortion in points' distribution, spline functions enabled very precise description of the surface, sufficient for mapping the shape of any surface objects.

4.2. Kriging interpolation

The first step in analysis was to construct empirical semivariogram for the residuals from the fitted polynomial trend of 3rd degree reflecting the global behavior (large scale trend) of the object under study. With regard to the continuity and regularity of test surface, the following semivariograms were taken into account: gaussian, cubic, sinehole effect (more on the theoretical models of semivariograms can be found in Journel, Huijbregts 2003, Armstrong 1998, Cressie 1993). In the process of model cross-validation, the best results were obtained for cubic semivariogram model (Fig. 6) and for the neighbourhood consisted of 42 points.

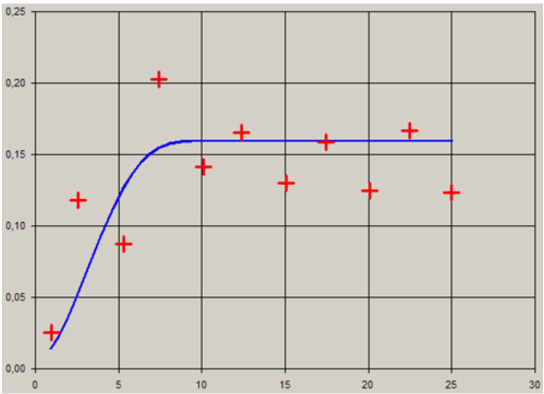


Fig. 6 Theoretical semivariogram model (continuous blue line) fitted to empirical values (red crosses)

Similarly as in the previous section the quantitative results that inform about the number of points with deviations within given accuracy intervals have been listed in Table 2 and also graphical presentation depicting the spatial distribution of deviations within given intervals is shown in Fig 7.

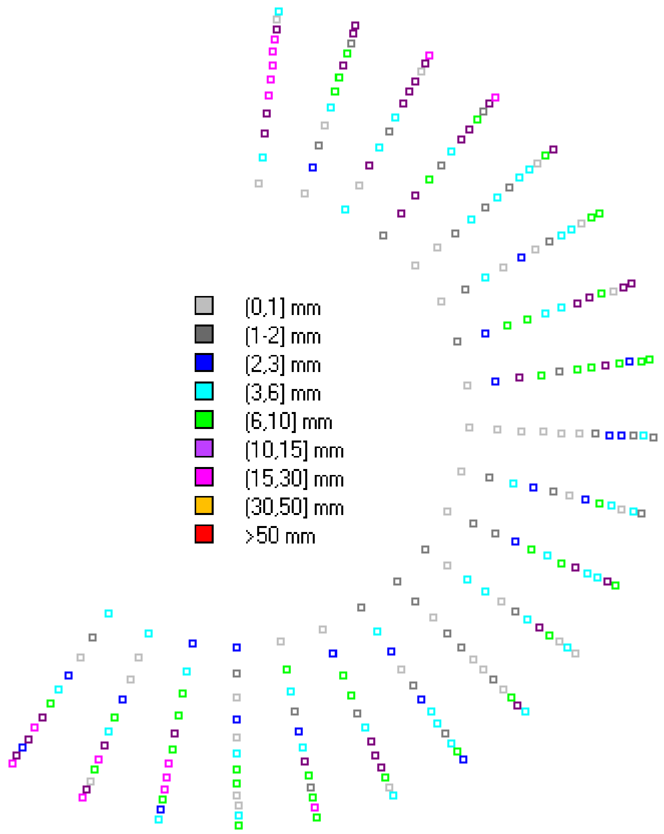


Fig. 7. Spatial distribution of deviations obtained by kriging interpolation

interval of deviations [mm]	number of points	contribution %
(0,1]	45	18,8
(1,2]	36	15
(2,3]	22	9,2
(3,4]	11	4,6
(4,5]	23	9,6
(5,6]	10	4,2
(6,7]	8	3,3
(7,8]	8	3,3
(8,9]	12	5
(9,10]	13	5,4
(10,12]	19	7,9
(12,14]	13	5,4
(14,16]	10	4,2
(16,18]	3	1,3
(18,20]	3	1,3
(20,25]	3	1,3
(25,30]	1	0,4
(30,35]	0	0
(35,40]	0	0
(40,60]	0	0
(60,80]	0	0

Tab. 2. The magnitude of deviations obtained by kriging interpolation

As it was expected, the worst accuracy in the method of kriging was obtained on the edges of the structure Fig. 7 what is caused by lack of neighbors. The only exception is the internal edge where in the first two strips of points deviations do not exceed 6 mm (except one point). In comparison to splines it is a very good result because in these places splines achieved the greatest deviations up to 80 mm. Although, the overall accuracy measured by number of points with deviations within given intervals is worse for kriging than for splines, kriging turned out to be more robust method to extreme deviations (max. dev. of 26 mm for kriging and 80 mm for splines). These features of the two methods directed the attention for combining them in order to obtain more accurate results.

5. CONCLUSIONS, SUGGESTIONS FOR FUTURE RESEARCH

The tests demonstrated the advantages and disadvantages to both methods of approximation. Spline functions offer very good quality of approximations of shell objects' surfaces (single mm) however, the relatively regular distribution of observations must be provided. Its lack results in rapid propagation of errors and leads to exceeding acceptable level of accuracy (obtained errors up to 80 mm). Kriging, in turn, gives a lower overall accuracy (compare tables 1 and 2) but is relatively stable for the entire object (maximum deviation of 26 mm). These observations lead the authors to the attempt of combining the two methods, in order to create a model reflecting their best features.

The idea is, to regularize the set of observation using kriging method and then to create a spline surface based on the so-prepared data. Regularization will only cover areas with lower regularity of observation, in order not to cause any deterioration of accuracy in the areas for which the spline functions offer a better accuracy than kriging.

Preliminary results obtained confirm expectations, i.e. a reduction of surface deformation by several times in areas of irregular distribution of observations.

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