Assessing the Use of ″Light″ Laser Scanners and the Monte Carlo Technique for the Documentation of Geometric Surfaces

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Key words: total station, scanning, geometric surface, Monte Carlo technique, fitting

SUMMARY

Modern constructions, as well as many other industrial products, have special geometric - defined surfaces according to their specifications.
Till today, the documentation of such surfaces was usually carried out by using the classic laser scanners, which unfortunately are very expensive to buy, heavy to transport at field and complicated in the procedure of the measurement.
About two years ago, a new generation of total stations with incorporated laser scanner capability has appeared in the market. Those total stations combine powerful laser EDMs with sophisticated software, which allow the user to collect scanning data at the field simultaneously with his classical survey measurements.
The so called *″light″ laser scanners*, are remarkably lightweight, definitely cheaper than the classic laser scanners and have user friendly software interface. For all the above reasons, they can today easily replace classical laser scanners in many applications.
This paper implies a standard procedure , which leads to the following results:

- A-priori estimation of the measured points' uncertainty by using the ″light″ laser scanners.
- Selection of the proper total station and the appropriate scanning distance according to the desired uncertainty result.
- Determination of the scanning parameters such as the scanning step (horizontally and vertically) and consequently the maximum number of points to be measured, as well as the a-priori standard error (σ_0) of a geometric surface adjustment.

For this purpose two methods are being combined: the Monte Carlo technique and the least square method.
An application on a difficult surface (satellite antenna) of an elliptic paraboloid was carried out, from which useful conclusions were drawn.

The new born ″light″ laser scanners proved to be adequate for the easier, quicker and cheaper documentation of geometric surfaces.

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1. INTRODUCTION

The last decade, modern total stations have obtained the capability to measure automatically in concrete directions a shaped pointcloud with a specific step on a given surface. This function is very convenient and many applications were performed over the last years. (Lambrou E., Pantazis G., 2006)
The evolution of this operation is a new generation of total stations, which are recently manufactured. These total stations, which incorporate a laser scanner, are very convenient in comparison to the classical laser scanners. These so called **″light scanners″**, are significantly lighter than big laser scanners, and moreover they do not need heavy external batteries or an own lap-top in order to take measurements. Their operation is almost the same as when they are measuring manually or automatically points. They are also servo or piezoelectric driven.
The scanning function of a total station can be performed in both short or long range mode, correspondly to the reflectorless measurements. Of course, different range provides different accuracy in the distance measurement.
In the table 1, two of the recently manufactured scanning total stations are presented with their main technical specifications.

Total Station	Distance accuracy (RL)	Angle accuracy	Max. Scan speed	Range (m)
Topcon IS	±5 mm	±0.3mgon	20 points/sec	<250
Trimble VX	±3 mm ±2 ppm	±0.3mgon	15 points/sec	<150

Table 1: Indicative ″light″ laser scanners models

In order to execute the scanning operation, an horizontal and vertical scanning step must be set. Given that thousands of points are not measured within a second, ’’light’’ laser scanners are mainly used for the scanning of geometric (mathematic) defined surfaces such as plans, spheres, cylinders, paraboloids, ellipsoids etc.
Many industrial products or other civil constructions have this kind of shapes, and it is indipsensable to check the corectness of those constructions at first stage, as well as their possible deformations in the future. Those deformations could be caused by physical phenomena such as earthquakes, crashes or soil erosion.
The aim of the present paper is to acquire the ability to predict the uncertainty of the coordinates’ (x, y, z) determination before scanning a surface. Additionally, the expected error σ_o of the fitting of the mathematic surface, namely the mathematic model, to the actual measured points is another very important element to find out.
Finally it would be interesting to know the appropriate scanning step according to the valid conditions of the measurerement in order to save time and labour.

The uncertainty of the x, y, *z* coordinates measured by such a method depends on the angle and distance measurement accuracy, which is given by the manufacturer. It also depends on the distance between the instrument and the surface which will be scanned. A parameter of major significance is the minimum scanning step that the instrument allows the user to set.

2. THE A-PRIORI SURFACE ADJUSTMENT

The control of geometric shaped equipment or constructions is made by comparing an ideal mathematic surface to the existing one at the concrete time. The mathematic equation which describes the surface is known as well as the characteristics of the instrument that will be used for its scanning. The following methodology uses the Monte Carlo technique and may be applied before the data collection.
At first, the interest is placed to the estimation of the mean uncertainty of the x, y, *z* coordinates determination. This uncertainty is calculated by equations 1 and 2 (Pantazis, et al, 2007), which come out from the fundamental geodetic problems for the coordinate's determination, when the law of propagation error is applied. The provided accuracies in the angle and distance measurement of the instrument must be known, as well as its distance from the surface to be scanned.

$$\sigma_{x_i} = \sigma_{y_i} \approx \pm\sqrt{\sigma_{D_i}^2 + \frac{D_i^2}{(\rho^{cc})^2}\cdot\left(\sigma_z^2 + \sigma_\alpha^2\right)} \tag{1}$$

$$\sigma_{z_i} \approx \pm\sqrt{\sigma_{D_i}^2 + D_i^2\cdot\left(\frac{\sigma_z}{\rho^{cc}}\right)^2} \tag{2}$$

Where

σ_{D_i} = the error of the distance measurement

σ_α, σ_z = the errors of the horizontal and vertical angles measurements respectively.

D_i = the distance between the instrument and the surface.

By using the Monte Carlo technique, the uncertainties σ_{xi}, σ_{yi}, σ_{zi} can be a-priori determined under the presupposition that the errors follow the normal distribution.
The Monte Carlo technique provides an estimation of the uncertainty following the next steps: Using a set of generated samples, the PDF for the value of the output quantity Y in equation (3) will be numerically approximated (Alkhatib, et al, 2009).

$$Y = f(Z_1, Z_2, ... Z_n) = f(Z) \tag{3}$$

Step 1: A set of random samples $z_1, z_2, \ldots, z_n$, which have n parameters, is generated from the PDF for each random input quantity $Z_1, Z_2, \ldots, Z_n$. The sampling procedure is repeated M times for every inserted quantity.

Step 2: The output quantities y will be then calculated according to the equation:

$$y^{(i)} = f(z_1^{(i)}, f_2^{(i)}, ..., f_n^{(i)}) = f(z^{(i)}) \tag{4}$$

Where $i = 1, ..., M$ are the generated samples of the random output quantity Y.

Step 3: Particularly relevant estimations of any statistical quantities can be calculated.

- the expectation of the output quantity

$$\hat{E}(f(z)) = \hat{E}(y) = \frac{1}{M}\sum_{i=1}^{M} f(z^{(i)}) \tag{5}$$

- the estimation of the variance of the output quantity

$$\hat{D}(y) = \frac{1}{M}\sum_{i=1}^{M}(f(z^{(i)}) - \hat{E}(f(z)) \cdot (f(z^{(i)}) - E(f(z))^{T} \tag{6}$$

Afterwards, these uncertainties can be used as the input quantities for the theoretical adjustment of the geometric surface, thus the a-priori σ_0 of the adjustment is estimated.

In other words the a-priori σ_0 of the points fitting to the geometric proper surface, considering that they are measured with the previous calculated uncertainties.

As it will be analyzed in paragraph 3, the Monte Carlo technique performs the adjustment by using the least squares method and the mathematic model of the surface. The calculation of the a priori σ_0 is independent from the scanning step, namely the total number of points. Figure 1 presents in a diagram the previous procedure.

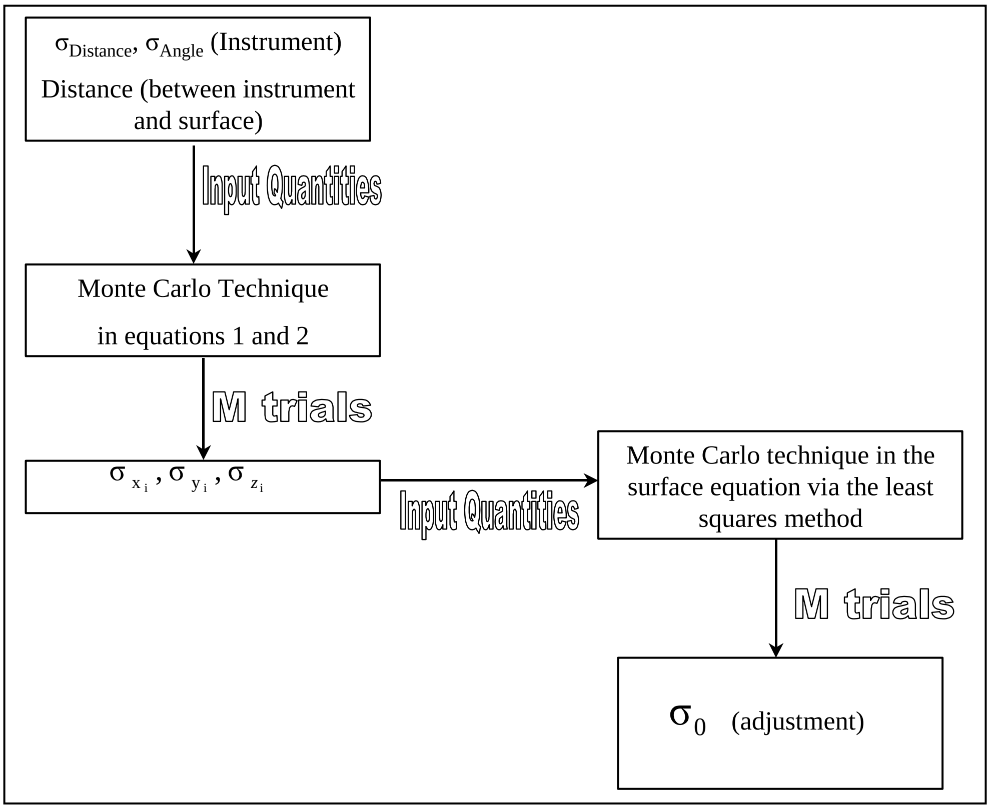


Figure 1. The a-priori estimation of the coordinates' uncertainties and the standard error σ_0 of the adjustment

Figure 2 illustrates the σ_x, σ_y, σ_z values as they are calculated by the Monte Carlo technique for four different angle accuracies, distance measurement with accuracy of ±3mm and scanning distances between 10m and 80m.

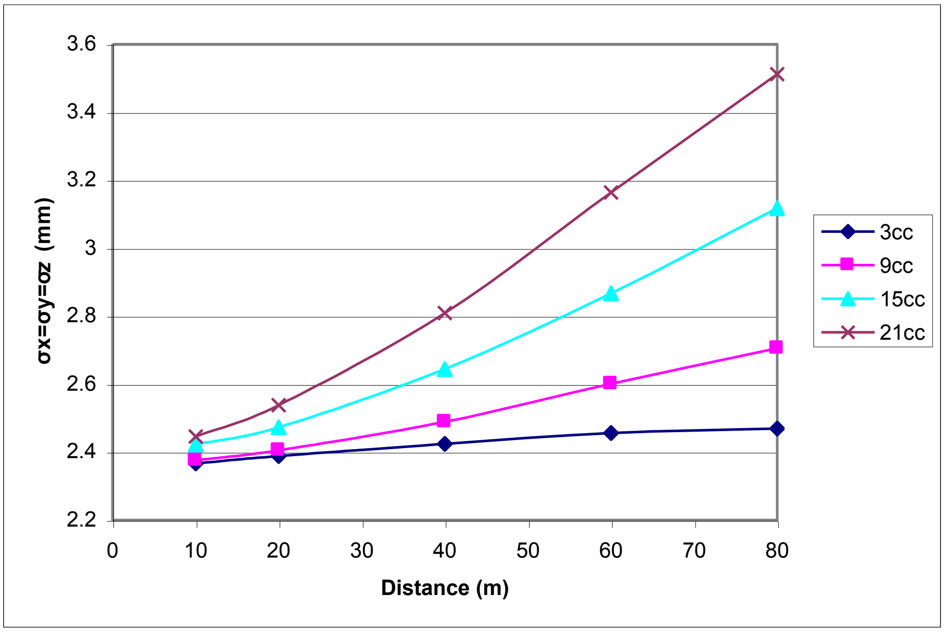


Figure 2. The σ_x, σ_y, σ_z values for different instruments, for distance measurement with an accuracy of ±3mm

The diagram in Figure 3, presents the results by applying the previous procedure for a plan, described by the equation 5x+8y-2z=0, considering that the total station provides an accuracy of ±3mm in the distance measurement.

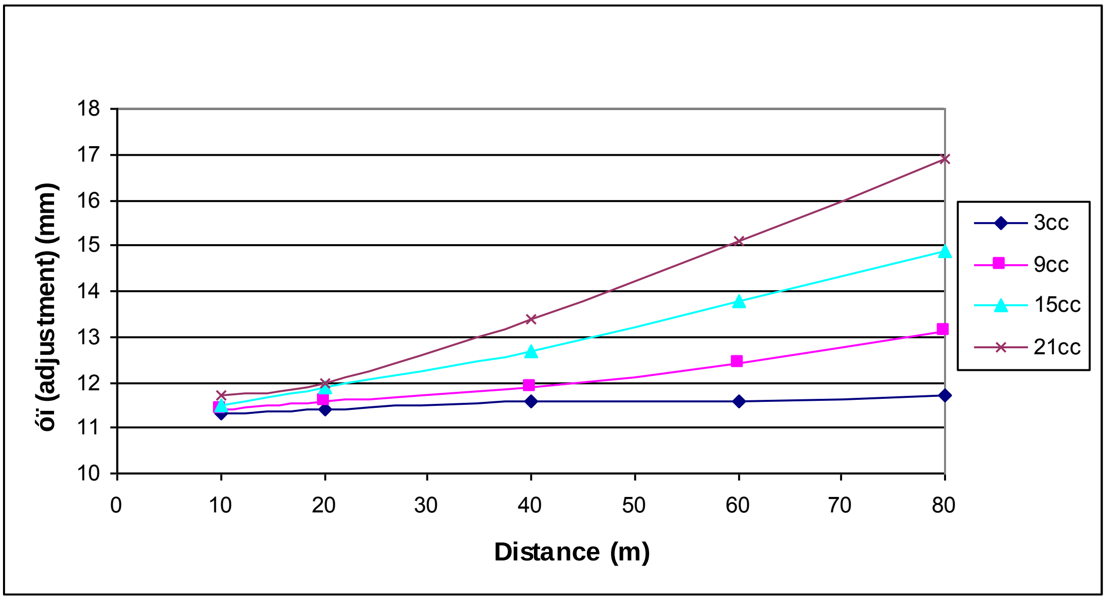


Figure 3. The a-priori standard error σ_0 for the adjustment of a plan

By using the procedure of the theoretical surface adjustment which was analyzed before, the selection of the proper total station can be done in accordance with the provided angles and distance measurement uncertainties and the scanning distance.

Another crucial question is how many points have to be scanned on a concrete surface, from a given distance by using the specific instrument.

As the a priori adjustment standard error σ_0 is detrermined, the minimum scanning step - namely the maximum number of points on the surface can be estimated.

Considering the same step, horizontally and vertically, every point could be measured by a step **s** equal to $\sigma_0 \cdot z_{95\%}$, where the $z_{95\%}$ is the coefficient of the normal distribution for confidence level 95%. A smaller step isn't useful as it would be within the error of the adjustment.

Thus, knowing the minimum necessary scanning step, the approximate number **n** of the points to be scanned may be estimated by the equation:

$$n = \frac{\text{Area}}{s^2} \qquad (7)$$

3. THE A - POSTERIORI SURFACE ADJUSTMENT

For the a posteriori surface determination, the least square method is being used. Usually, the unknown surface is expressed by a linear or no linear function of the calculated x, y *z*, coordinates of the surface. The unknown parameters are the coefficients a_i of the surface's equation.

In these cases a linearisation of the observation

$$A \cdot \hat{x} = l + u \qquad (8)$$

where

$$A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1m} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2m} \\ \vdots & \cdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nm} \end{bmatrix},\ \hat{x} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}, l = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_m \end{bmatrix} \qquad (9)$$

By the above equations system, the vector of the unknown coefficients a_i of the surface equation can be determined as follows:

$$\hat{x} = \left(A^T \cdot A\right)^{-1} \cdot A^T \cdot l$$

Additionally, the a posteriori standard error $\hat{\sigma}_0$ of the adjustment and the corresponding variance – covariance matrix V_x are being calculated.

$$V_x = \begin{bmatrix} \sigma^2_{a_1} & & & \\ & \sigma^2_{a_2} & & \\ & & \ddots & \\ & & & \sigma^2_{a_m} \end{bmatrix}$$

The diagonal elements of this matrix are the variances of the surface's equation coefficients a_i.

Continuous statistical checks may be applied in order to prove the validation of the adjustment.
The values of the coefficients a_i should be checked for their statistical significance for confidence level 95%, by using the following equation:

$$\acute{o}_{a_i} \cdot z_{95\%} \leq a_i \tag{10}$$

If equation 10 is valid for all the coefficients then the surface is acceptable.
Another criterion which will be applied in order to help us decide if the measured points fit to the concrete surface is the comparison of the a posteriori standard error $\hat{\sigma}_0$ of the adjustment with the a priori one, which came out by the Monte Carlo technique. Namely the following equation can be formed:

$$\acute{o}_{a\text{-posteriori}} \leq \acute{o}_{a\text{-priori}} \cdot z_{95\%} \tag{11}$$

If the equation 11 is valid then the measured points belong to the geometric-mathematic model and hence the surface has no manufacture errors and it was constructed in accordance to the specifications.
On the contrary if the equation 11 is not valid, the residual of all the points must be analyzed. By this check, the deformed areas of the surface can also be determined.

4. APPLICATION

4.1 The scanning surface

The aim of the application is the assessing of the credibility of a 3D model provided by the modern ″light″ laser scanners. For this purpose the scan of a satellite antenna (fot.1) was carried out. It is known that the shape of the antenna is an elliptic paraboloid (figure 4), whose surface is expressed by the following equation:

$$\frac{(x_i - x_0)^2}{a^2} + \frac{(y_i - y_0)^2}{b^2} = \frac{1}{c} \cdot (z_i - z_0) \tag{12}$$

where

- a, b, c are the semi axes of the elliptic paraboloid solid surface
- x_0, y_0, z_0 are the coordinates of the surface's center
- x_i, y_i, z_i are the coordinates of each measured point i on the surface

The equation 12 is not linear. After its linearalization using the Taylor method the following equation comes out:

$$-\frac{2 \cdot (x_i - x_0)^2}{a^3} \cdot \delta a - \frac{2 \cdot (y_i - y_0)^2}{b^3} \cdot \delta b - (z_i - z_0) \cdot \delta c - \frac{2 \cdot (x_i - x_0)}{a^2} \cdot \delta x_0 - \frac{2 \cdot (y_i - y_0)}{b^2} \cdot \delta y_0 + \frac{1}{c} \cdot \delta z_0 = 0 \tag{13}$$

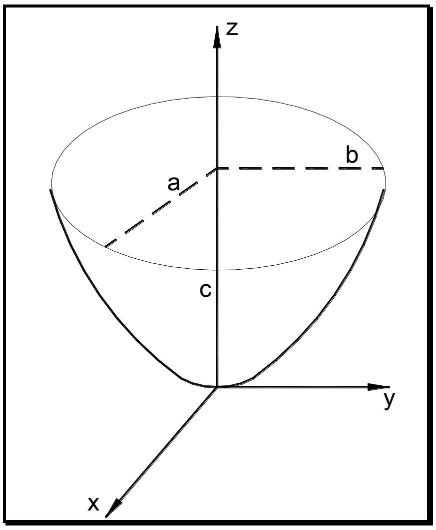
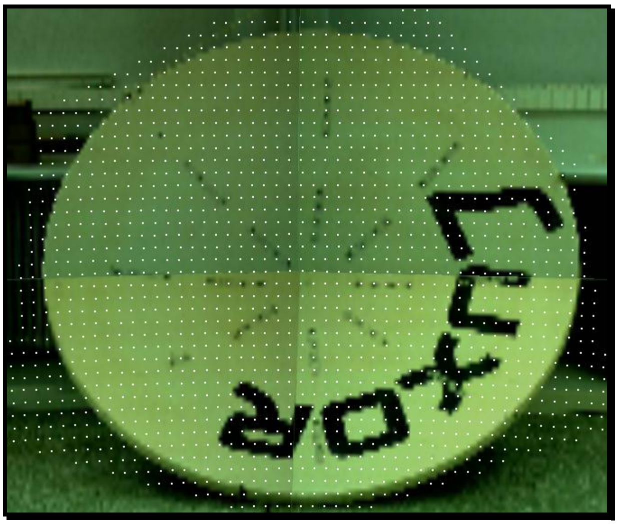


Figure 4. The elliptic paraboloid



Fot. 1. The satellite antenna and measured points

4.2 The a-priori parameters determination

For the a priori determination of the interesting parameters ($\sigma_{x_i}, \sigma_{y_i}\ \sigma_{z_i}, \sigma_0$) the above analyzed methodology is being applied.

Figure 5 presents the a priori standard error σ_0 of the adjustment of the scanning procedure,for an elliptic paraboloid surface when expressed by equation 12. Considering that it is measured by different instruments and to different distances, $x_0 = y_0 = z_0 = 0$m, which is defined in an arbitrary reference system and additionally a = 0.94m, b = 0.96m and c = 0.3m.

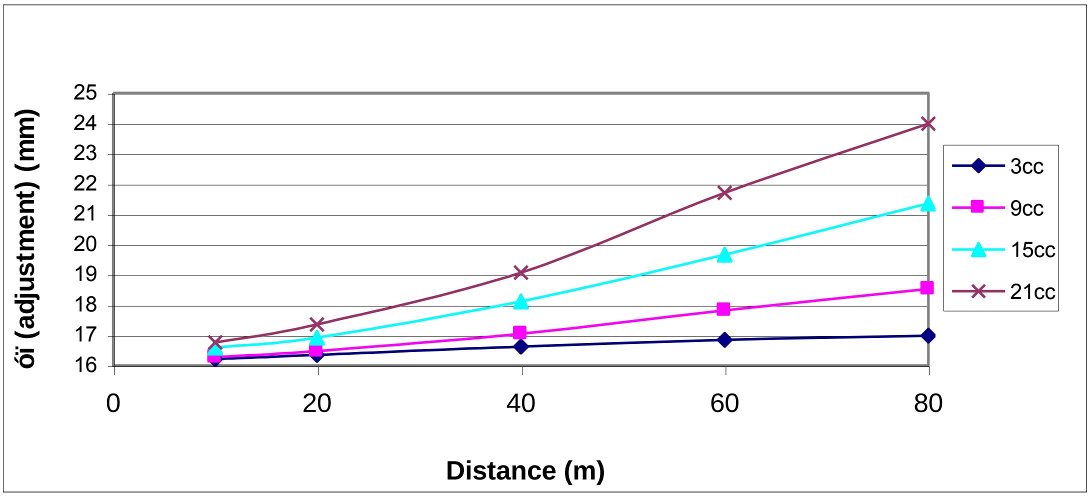


Figure 5. The a-priori standard error σ_0 of the adjustment for an elliptic paraboloid surface.

As a result, the best adjustment for standard error σ_0 can be reached for the surface of the elliptic paraboloid, using a total station with $\pm 3^{cc}$ angle accuracy and for scanning distance of about 10m. In this case the adjustment's standard error is equal to 1.6cm. Therefore the minimum scanning step will be $1.6 \cdot z_{95\%} = 3.2\text{cm}$. Consequently 1000 points will be measued for every square meter.

4.3 The a-posteriori surface determination

Taking into consideration the a priori analysis of the scanning, the Trimble VX total station was decided to be used for this application.
The provided uncertainties by this instrument are $\pm 3^{cc}$ for the directions and $\pm 3\text{mm} \pm 3\text{ppm}$ for the distance measurements respectively (Trimble, 2008).
The scanning was carried out using a scanning step of 4cm horizontally and vertically, which is approximately equal to the a priori analysis. About 2600 points were measured (including the useful ones). The points, which are situated outside the antenna's body, were removed.
The coordinates x, y and *z* of the measured points are calculated in a local arbitrary reference system.
In the case that there were measured n points on the antenna's body, n equations as the equation 13 are formed.
Each one has five unknown parameters (δa, δb, δx_0, δy_0, δz_0). The freedom degree of the system is r = n-5. These parameters and their uncertainties (table 2) are determined by the least square method as it was mentioned before, using the blue print coordinates of the unknown parameters as their temporary values.

a(m)	σ_a (mm)	b(m)	σ_b (mm)	x_0 (m)	σx_0 (mm)	y_0 (m)	σy_0 (mm)	z_0 (m)	σz_0 (mm)
0.951	±0.3	0.965	±0.3	0.001	±0.0001	-0.019	±0.0002	0.001	±0.0001

Table 2: The values of the parameters a, b, x_0,y_0,z_0 and their standard errors

The a-posteriori standard error of the adjustment resulted to be ±12mm. The equation 11 is valid, namely $\sigma_{\text{a-posteriori}} = \pm 12\text{mm} < \sigma_{\text{a-priori}} \cdot z_{95\%} = \pm 16 \cdot 1.96 = \pm 31\text{mm}$.
Hence the theoretical analysis is confirmed.
Also the values of the parameters a, b, c, x_0, y_0, z_0 are checked and found that they are statistical significant as the equation 10 is valid.

5. REMARKS AND CONCLUSIONS

The presentation of a complete methodology for the determination of the adjustment parameters for the geometric surface documentation is the aim of this paper. This work is focused mainly in the a-priori estimation of the standard error of the adjustment in relation to the used total station as ″light″ laser scanners but also in the selection of the proper scanning step namely the needed points to be scanned.
The conclusions could be summarized at the following:

- The a priori standard error σ_0 of the points' adjustment, which belong to a specific surface is strongly influenced by the accuracy that the total station provides and can estimated by using the Monte Carlo technique.

- Knowing the number of points which are necessary to be captured and the desired σ_0 of the adjustment, the user can have a clear idea of what he needs to collect at the field.
- The comparison between the a-priori and the a-posteriori σ_0 of the adjustment can document that the measured surface is constructed according to its specifications.
- The Monte Carlo technique proved to be a very useful tool for the a-priori determination of the measurements' uncertainty as well as the standard error σ_0 of the adjustment.
- The development of total stations, with laser scanner capability, gives the opportunity for a more economical procedure of scanning geometric surfaces compared with the real laser scanner.
- These instruments are more convenient to the data processing compared to the laser scanners as not huge computer needed and more easy to use as they are lighter and have the same on board software as the conventional surveys.

REFERENCES

Alkhatib, H., Neumann, I., Kutterer, H. (2009). "Uncertainty modeling of random and systematic errors by means of Monte Carlo and fuzzy techniques". Journal of applied geodesy 3, 67-79.

Lambrou E., Pantazis G. (2006). "A new geodetic methodology for the accurate documentation and monitoring of inaccessible surfaces". Proceedings of ″3rd IAG symposium on Geodesy for Geotechnical and Structural Engineering and 12th International FIG symposium on deformation measurements", Baden, Austria.

Pantazis, G., Lambrou, E., Nikolitsas, K., (2007)."The relationship of the measurements uncertainty and the result uncertainty in geodetic applications ". Proceedings of ″2nd National Metrological Conference Hellenic Laboratory Union, Thessalonica,

Trimble Navigation (2008), Users manual for Trimble Vx.

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