Theoretical and Empirical Minimum Detectable Displacements for Deformation Networks

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Key words: GPS Network, Horizontal Control Network, Displacement Ellipse, Theoretical Minimum Detectable Displacement, Empirical Minimum Detectable Displacement

SUMMARY

In deformation analysis, statistical methods have been used to detect displaced point(s) and to identify the characteristic feature of object with a high of probability. These methods can sometimes have wrong results; that's why the reliability of the method should be known. However; to measure the reliabilities of the methods, it is required to know actual displaced point(s) before the analysis. Since it is impossible in practice, displacements of points are simulated. Displacement ellipse of point have been used to determine the magnitude of minimum detectable displacements of the point. However, the magnitudes and directions of semi-minor and semi-major axes are different, corresponding displacement circle can be used to calculate minimum detectable displacement. Displacement circle is an empirical method. Also, minimum detectable displacement based on the non-centrality parameter is also important for the sensitivity of a deformation network which is characterised its efficiency to detect displacements in the area covered by the network. This method is called theoretical method. In this study, we have investigated for answering the question "which method is more realistic?". Both empirical and theoretical minimum detectable displacements have been calculated for horizontal control network and GPS network. At the same time, an empirical method based on global test for minimum detectable displacement has been estimated. The theoretical minimum detectable displacements changes for different power of the test value (e.g. 70%, 80%). It has been shown that the theoretical minimum detectable displacements converges to empirical minimum detectable displacements when the power of test is %70.

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1. INTRODUCTION

Being the cause of massive losses of life and property, deformation analysis is of vital importance in determining the movements originated from tectonics, landslides or man-made buildings (dams, bridges etc.). Conventional vertical, horizontal or three dimensional networks as well as GPS networks are established to control behaviour of these movements periodically. The main goal is to answer the question that whether any displacement occur or not. Various studies have been performed regarding deformation analysis (Pelzer, 1971; Caspary et al, 1983; Koch, 1985; Niemeier, 1985; Schaffrin, 1986; Caspary, 1988; Caspary et al., 1990; Schaffrin & Bock, 1994; Betti et al., 1999; Snow & Schaffrin 2007; Cai et al., 2008).

Accuracy, reliability and sensitivity should be taken into consider in the geodetic deformation network design phase (Cooper, 1987). Accuracy refers to quality of the network. Reliability is the meaning of effect of the undetectable outliers on deformation analysis (Aydin & Demirel, 2005). Detectability of expected displacements and deformations at the designated network corresponds to sensitivity (Even-Tzur, 2006). To measure the efficacy of the Conventional Deformation Analysis (CDA), prior knowledge about which point has been displaced is needed. In practice, it is impossible to obtain this information; hence the points' displacements have been simulated for various studies (Hekimoglu et al., 2010; Duchnowski & Wisnievski, 2014; Nowel & Kaminski, 2014; Velsink, 2015; Nowel, 2015).

There are three main factors affect the sensitivity of deformation network; network design, session duration (for GPS measurements) and accuracy of measuring instrument. Since the accuracy of estimated coordinates and their (co)variances depends on the all these factors, the network should be capable of detecting expected deformation magnitude. To determine the detectable displacement magnitude, the ellipses of the points are used as criterion. Displacement area is identified with the displacement ellipses. Standardized expected displacement ellipses of the points in the network can be used. Displacement ellipses are the global confidence region. If the magnitude of the estimated displacement vector lies entirely within the properly scaled expected point displacement ellipse, then there is no significant movement of the point at the chosen significance level α (Cooper, 1987).

For the simulation, one main problem is how to generate the displacement magnitude in the deformation networks. In this study, our aim is to specify Theoretical and Empirical Minimum (Minimal) Detectable Displacements (TMDD, EMDD). These magnitudes have been used to compare with each other for Horizontal Control Network (HCN) and also GPS network. For this purpose; 3 points (A, B and C) of HCN and 3 points of GPS network (OBC1, OBC2 and OBC3) were chosen to calculate one TMDD and two EMDD' s magnitudes.

Overview of this paper: After a review of the conventional deformation approach in Sect. 2, the paper gives in Sect. 3 a description of the EMDD. Sect. 4 presents TMDD and simulated networks have shown in Sect. 5. Conclusions are given about the results of the experiments in Sect. 6.

2. CONVENTIONAL DEFORMATION ANALYSIS

The typical CDA compares coordinate differences in a geodetic network between two different observation epochs. If H_0 hypothesis rejected as expected movements by statistical tests, these differences are interpreted as being the "displacements." Conventionally, the object and its surrounding are characterised by discrete points in deformation monitoring network and only the geometrical changes of the object are modelled. The monitoring network is adjusted as a free network for each epoch and coordinate differences between two different observation epochs are compared. The first step of the CDA is global congruency test. The model which suggests that the coordinates of corresponding points between two epochs do not change differently from expectations is formed as the null hypothesis (Welsch & Heunecke 2001)

$$H_0: [-\mathbf{I} \quad \mathbf{I}] \begin{bmatrix} E(\hat{\mathbf{x}}_1) \\ E(\hat{\mathbf{x}}_2) \end{bmatrix} = \mathbf{0} \tag{1a}$$

$$H_1: [-\mathbf{I} \quad \mathbf{I}] \begin{bmatrix} E(\hat{\mathbf{x}}_1) \\ E(\hat{\mathbf{x}}_2) \end{bmatrix} \neq \mathbf{0} \tag{1b}$$

where E() stands for expectation, $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$: The vectors of estimated coordinates of the first and the second epochs, $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ are estimated by free network adjustment as following equations:

$$\hat{\mathbf{x}}_1 = \mathbf{Q}_{\hat{\mathbf{x}}1\hat{\mathbf{x}}1}\mathbf{A}_1^{\mathbf{T}}\mathbf{P}_1\mathbf{l}_1,\ \mathbf{Q}_{\hat{\mathbf{x}}1\hat{\mathbf{x}}1} = \mathbf{N}_{11}^{+} = (\mathbf{A}_1^{\mathbf{T}}\mathbf{P}_1\mathbf{A}_1)^{+} \tag{2}$$

$$\hat{\mathbf{x}}_2 = \mathbf{Q}_{\hat{\mathbf{x}}2\hat{\mathbf{x}}2}\mathbf{A}_2^{\mathbf{T}}\mathbf{P}_2\mathbf{l}_2,\ \mathbf{Q}_{\hat{\mathbf{x}}2\hat{\mathbf{x}}2} = \mathbf{N}_{22}^{+} = (\mathbf{A}_2^{\mathbf{T}}\mathbf{P}_2\mathbf{A}_2)^{+} \tag{3}$$

$$\Omega = \mathbf{v}_1^{\mathbf{T}}\mathbf{P}_1\mathbf{v}_1 + \mathbf{v}_2^{\mathbf{T}}\mathbf{P}_2\mathbf{v}_2 \tag{4}$$

$$s_0^2 = \frac{\Omega}{f}, f = f_1 + f_2 \tag{5}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ **:** Design matrices of the first and second epochs, respectively, if the configuration of the network in different epochs is not changed $\mathbf{A}_1$=$\mathbf{A}_2$, $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ **:** Vectors of estimated coordinates of the first and the second epochs, $\mathbf{P}_1$ and $\mathbf{P}_2$ **:** Weights matrices at the first and the second epochs, $\mathbf{l}_1$ and $\mathbf{l}_2$ **:** Observations vectors of the first and the second epochs, $\mathbf{v}_1$ and $\mathbf{v}_2$ **:** Residual vectors of the first and the second epochs, $\mathbf{f}_1$ and $\mathbf{f}_2$ **:** Degrees of freedoms of the first and the second epochs, respectively.

Then, the influence of the null hypothesis on the Least Square Estimation (LSE), in the absence of correlations between epochs, results in (Pelzer 1971; Niemeier 1985; Koch 1985; Cooper 1987)

$$\mathbf{d} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1 \tag{6}$$

$$\mathbf{Q}_{\mathbf{dd}} = \mathbf{Q}_{\hat{\mathbf{x}}1\hat{\mathbf{x}}1} + \mathbf{Q}_{\hat{\mathbf{x}}2\hat{\mathbf{x}}2} \tag{7}$$

$$R = \mathbf{d}^{\mathbf{T}}\mathbf{Q}_{\mathbf{dd}}^{+}\mathbf{d} \tag{8}$$

where $\mathbf{d}$: difference vector of estimated coordinates, $\mathbf{Q_{dd}}$: Cofactor matrix of $\mathbf{d}$, $\mathbf{Q^{+}_{dd}}$: Pseudoinverse of the cofactor matrix.

$T = \frac{R}{hs_0^2} \sim F_{h,f,a}$, under H_0, assuming normally distributed observations errors,

h is the rank of the matrix $\mathbf{Q_{dd}}$, a : Error probability, s_0^2 : Estimated variance component in the absence of the null hypothesis.

If $T > F_{h,f,a}$, then the null hypothesis is rejected. Thus, the difference in the coordinates between two epochs is interpreted as the result of an unexpected displacement (Niemeier, 1985; Welsch & Heunecke 2001).

3. EMPIRICAL MINIMUM DETECTABLE DISPLACEMENT

The lengths and directions of the semi-axes of the expected displacement ellipses are different for each point in the network. Thus these ellipses are not convenient for a simulation which generates the magnitude of the displacement. To compare the successes of the CDA in different cases, a circle is chosen instead of an ellipse. The circle whose area is equal to the area of the expected displacement ellipse could be chosen as given in Fig. 1, so that the total area of the positive part is equal to the total area of the negative part. The radius of the displacement circle may be estimated as a mean value from the different radius for a given network (Hekimoglu et al., 2010). First EMDD magnitude calculated from the formulas shown as in the following section.

3.1 Empirical Technique 1: Using Displacement Ellipse

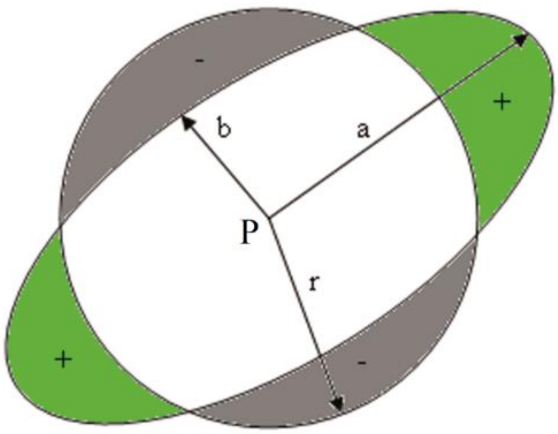

Fig. 1. Expected displacement ellipse and corresponding displacement circle of Point P (a and b are the semi-major and semi-minor axes of the expected displacement ellipse) (Hekimoglu et al., 2010).

The formulas for the elements of the rescaled expected displacement ellipse and the corresponding displacement circle are given as follows:

$$\text{int} = q_1 r < |z_i| < q_2 r,\ q_1 \geq 1,\ q_1 \neq q_2,\ q_1 < q_2 < 3 \tag{9}$$

$$a = \sigma_0 \sqrt{\lambda_1 \chi^2_{2,1-\alpha}} \tag{10}$$

$$b = \sigma_0 \sqrt{\lambda_2 \chi^2_{2,1-\alpha}} \tag{11}$$

$$\lambda_1 = \frac{Q_{dxixi} + Q_{dyiyi} + w}{2} \tag{12a}$$

$$\lambda_2 = \frac{Q_{dxixi} + Q_{dyiyi} - w}{2} \tag{12b}$$

$$w = \sqrt{(Q_{dxixi} - Q_{dyiyi})^2 + 4Q^2_{dxiyi}} \tag{13}$$

$$r = \sqrt{a_{mean} b_{mean}} \tag{14}$$

where Q_{dxixi} and Q_{dyiyi} = elements of the respective submatrix of the cofactor matrix $\mathbf{Q_{dd}}$ from Eq. 7, which belongs to the ith point, λ_1 and λ_2 = Semi-major and semi-minor axes of the standardized expected displacement ellipse, σ_0^2 = A priori variance component, $\chi^2_{2,a}$ = a - fractile of the χ^2_2-distribution for 2 degrees of freedom, r = Radius of the corresponding displacement circle.

The value of a is chosen here as 0.001 so the stochastic effect is reflected almost entirely in the simulated displacement magnitude.

3.2 Empirical Technique 2: Step Approach Testing

Second EMDD magnitude was obtained by step approach testing. This technique estimates the minimum magnitude of displacement for different directions depending on the global congruency test that is given in Eqs. (6), (7) and (8). To reach the final step quicker, the first value has chosen randomly which can both detectable and reflects the stochastic model of network. Then first value was increased or decreased 0.1 mm to get the minimum displacement magnitude.

3.2.1 Simulation of Displacement Vector

To use in deformation analysis, the random errors were generated differently for each epoch. These vectors were added to free-error measurements. Only random errors ($\mathbf{e_1}$) were added to 1st epoch measurements. For 2nd epoch; both random errors ($\mathbf{e_2}$) and displacements of the points ($\mathbf{z}$) were added to free-error measurements. 1st and 2nd epoch measurements given as follows (Hekimoglu et al., 2010):

$$\mathbf{l_1} = \bar{\mathbf{l}} + \mathbf{e_1} \tag{15}$$

$$\mathbf{l_2} = \bar{\mathbf{l}} + \mathbf{e_2} + \mathbf{Az} \tag{16}$$

Where $\mathbf{e_1}$ and $\mathbf{e_2}$: normally distributed random error vectors, $\mathbf{A}$: Coefficient matrix, $\mathbf{z}$: Horizontal displacement vector and $\bar{\mathbf{l}}$: the observation vector without random errors.

$$\mathbf{z} = [z_{1x} \quad z_{1y} \quad z_{2x} \quad z_{2y} \quad \cdots \quad z_{ux} \quad z_{uy}] \tag{17}$$

z_x : The projection of displacement vector **z** to x axes of the displacement, z_y : The projection of displacement vector **z** to y axes of the displacement.

4. THEORETICAL MINIMUM DETECTABLE DISPLACEMENT

The minimal detectable value can be used as a different approach to obtain the test value. Let $\mathbf{d_k}$ and σ_0^2 be the expected displacement vector and a priori variance of unit weight, respectively. To determine these displacemets the alternative hypothesis must be accepted as shown in the Eq. 1b.

Then the test value is non-central F distribution and its eccentricity parameter is given as following equation:

$$\lambda = \frac{\mathbf{d_k^T Q_{dd}^+ d_k}}{\sigma_0^2} \tag{18}$$

The eccentricity parameter for specific value, power of test $\gamma = 1 - \beta$, error probability α and degrees of freedom (h,$f = \infty$), is compared with the threshold value λ_0. If $\lambda > \lambda_0$ it is inferred that the deformation network is sensitive. This comparison process called as sensitivity analysis (Caspary et al., 1983; Niemeier, 1985; Cooper, 1987; Even-Tzur, 2006; Aydin et al., 2004).

To determine the minimal detectable displacement magnitude of the points of network vector **g** that consists of movement direction is generated. For the two dimensional network the vector **g** is as

$$\mathbf{g} = [\cos t_1 \sin t_1 \cos t_2 \sin t_2 \ldots \cos t_p \sin t_p]^T \tag{19}$$

where p : The number of points, in vector **g** the components of the points which are considered as undisplaced is assumed as “0”**.** Thus the deformation vector is as

$$\mathbf{d_k} = \mathrm{a}\mathbf{g} \tag{20}$$

where a : The scale factor and it is computable. Then, by substituting Eq. (20) into Eq. (18) the equation has formed as

$$\lambda = \frac{\mathrm{a}^2}{\sigma_0^2} \mathbf{g^T Q_{dd}^+ g} \tag{21}$$

Then, by considering the inequation of $\lambda > \lambda_0$ with Eq. (21) the Eq. (22) is obtained as follows:

$$\frac{\mathrm{a}^2}{\sigma_0^2} \mathbf{g^T Q_{dd}^+ g} > \lambda_0 \tag{22}$$

From Eq. (22) the minimum value of “a” can be calculated as follows:

$$\mathrm{a}_{min} = \sigma_0 \sqrt{\frac{\lambda_0}{\mathbf{g^T Q_{dd}^+ g}}} \tag{23}$$

From the Eq. (20) the minimum detectable displacement magnitude can be obtained at any desired point in any direction by using a_{min}.

5. SIMULATED NETWORKS

5.1 Horizontal Control Network

Horizontal Control Network was simulated shown as Fig. 2. The number of observations and the number of point are 72 and 9, respectively. The degrees of freedom is 48 (by taking 9 orientation unknowns into account) with regard to free network adjustment. The lengths of the distance measurements are varied approximately between 3 and 7 km. Random error vectors were generated from a normal random error generator. The random errors e_{jk} come from the different normal distributions $N(\mu = 0, \sigma_d^2)$, where σ_d was chosen to be the same for the direction measurements at each point, such as ± 0.3 mgon and σ_d = $\pm$3+2*ppm* mm for the distance measurements.

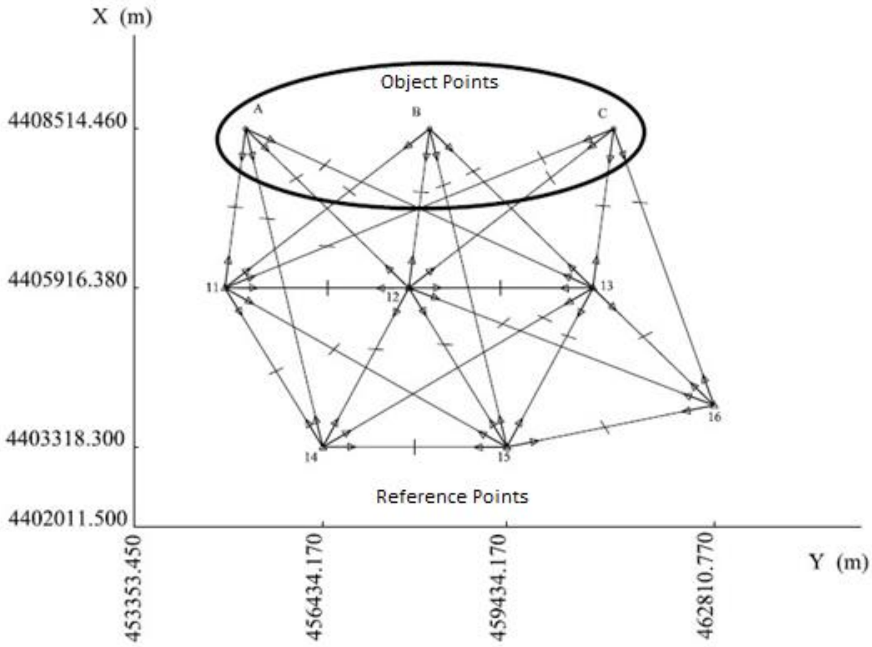

Fig. 2. Horizontal Control Network (Erdogan 2011, Hekimoglu et al., 2010).

The two epochs' geometries are the same; so the weight matrices of the observations are equal to each other for the two epochs as follows (assuming no correlation between epochs):

$$\mathbf{P} = \begin{bmatrix} \mathbf{P_d} & \mathbf{0} \\ \mathbf{0} & \mathbf{P_s} \end{bmatrix} \tag{24}$$

Where $\mathbf{P_d} = diag(\sigma_0^2/\sigma_{d1}^2, \sigma_0^2/\sigma_{d2}^2, \ldots, \sigma_0^2/\sigma_{dm_1}^2)$ (for direction measurements), $\mathbf{P_s} = diag\left(\sigma_0^2/\sigma_{s1}^2, \sigma_0^2/\sigma_{s2}^2, \ldots, \sigma_0^2/\sigma_{sm_2}^2\right)$ (for distance measurements), σ_0^2 : The variance of unit weight is chosen as 0.009 mgon^2, m_1 and m_2 : The number of direction measurements and the number of distance measurements, respectively, all assumed to be uncorrelated.

5.2 GPS Network

Deformation network has 3 points which were established at the Davutpasa Campus of Yildiz Technical University, 4 points from TUSAGA-AKTIF (SARY, KABR, SLEE, ISTN) network and 2 points from IGS network (ISTA, TUBI) as shown in Fig. 3. Thirty two baselines were observed in the network. Approximate baseline distances of the points used in the process stage with reference to ISTA varying from 14.2 to 99.8 km.

The network were processed at Bernese v5.0 software with observing points OBC1, OBC2 and OBC3 (Dach et al., 2007). 7 hours measurements were considered at processing stage. The optimal values were used as stated in Bernese v5.0 guide. Precise GPS orbits and Earth rotation parameters (EOP) were obtained from IGS data center (International GNSS Service). Approximate coordinates were calculated for new points and receiver errors by one point positioning with ionosphere free L3 frequency.

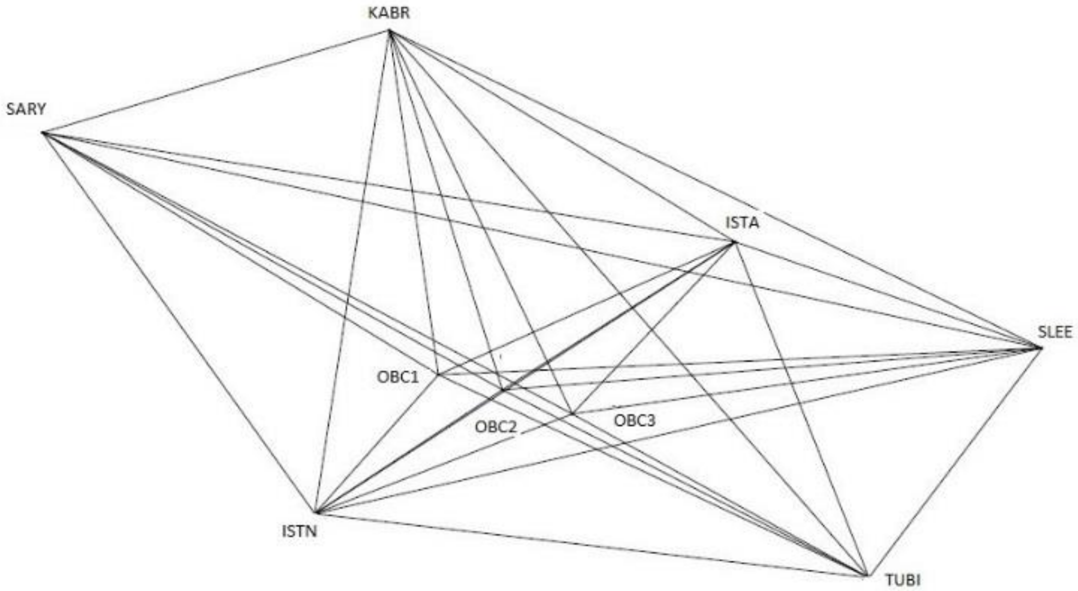

Fig. 3. GPS Network (Erdogan 2011, Erdogan & Hekimoglu 2014)

Cycle slips and outliers were checked by generating triple differences. Then, outliers were removed and cycle slips were corrected or initial phase ambiguity parameter were added to cycle slips which could have not corrected. Then using QIF (Quasi Ionosphere Free) technique initial phase ambiguity parameters were solved. Tropospheric zenith delay was disposed using Saastamoinen model. To use for defining the datum ISTA, which is IGS point and computed by SOPAC, was chosen at the datum 2005.0 and epoch 2010.835.

Obtained three-dimensional Cartesian coordinates were transformed to local coordinate (Topocentric coordinate) system. Since the applied GPS measurement complied with the criteria in the paper written by Eckl et al., 2001, the weight matrix was obtained by using the standard deviations given in this article.

6. RESULTS OF EXPERIMENTS

6.1 Results for Horizontal Control Network

The components of displacement circle were obtained for points A, B and C which are assumed as displaced and radius was calculated as r = 29.9 mm. Then, the EMDD magnitudes were also calculated, using the 2nd Empirical Technique: Step Approach test. To compare the magnitude with theoretical approach the minimum detectable displacements were computed at 40 different directions from Eqs. (19) - (20) with $\alpha = 0.05$ and $\beta = 0.20$. The displacements magnitudes were obtained for $\alpha = 0.05$ and $\beta = 0.30$ as well (Fig. 4, Fig 5, Fig. 6).

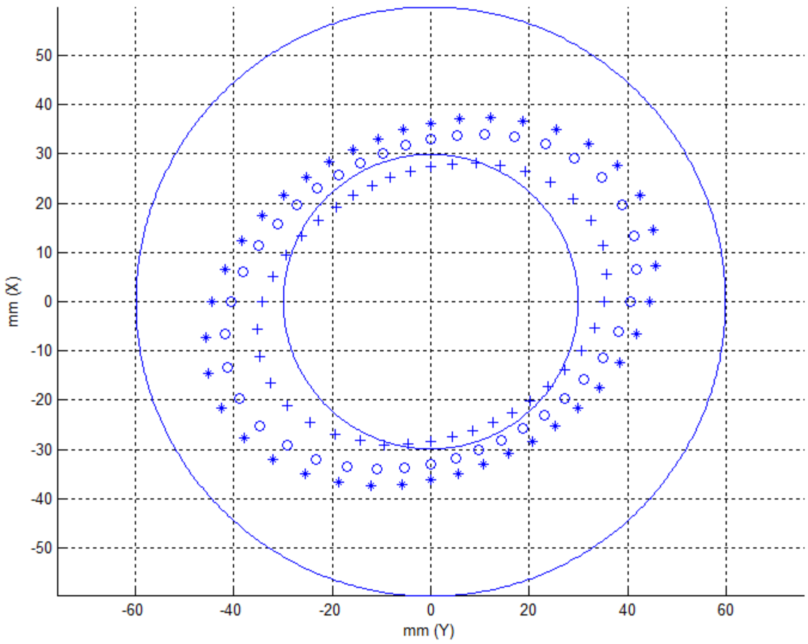

Fig. 4. TMDD and EMDD magnitudes for point A

"+" refers to displacement magnitudes which were obtained through 2nd Empirical Technique: Step Approach Testing whose minimum detectable displacements magnitudes of each points were calculated and shown for at 10 grade intervals.

"*" and "o" shows the TMDD magnitudes when $\alpha=0$, $\beta=0.20$ and $\alpha=0$, $\beta=0.30$, respectively.

The field between two circles at Fig. 4, Fig. 5, Fig. 6 are corresponds to (r, 2r) interval that is obtained from 1st Empirical Technique: Using Displacement Ellipse shown as sect. 3.1. This algorithm was used for calculating the radius of circles.

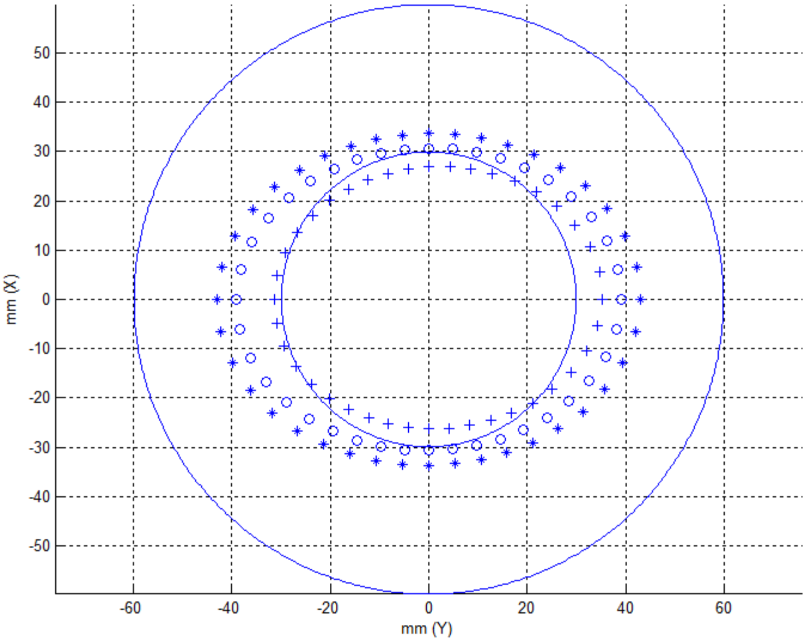

Fig. 5. TMDD and EMDD magnitudes for point B

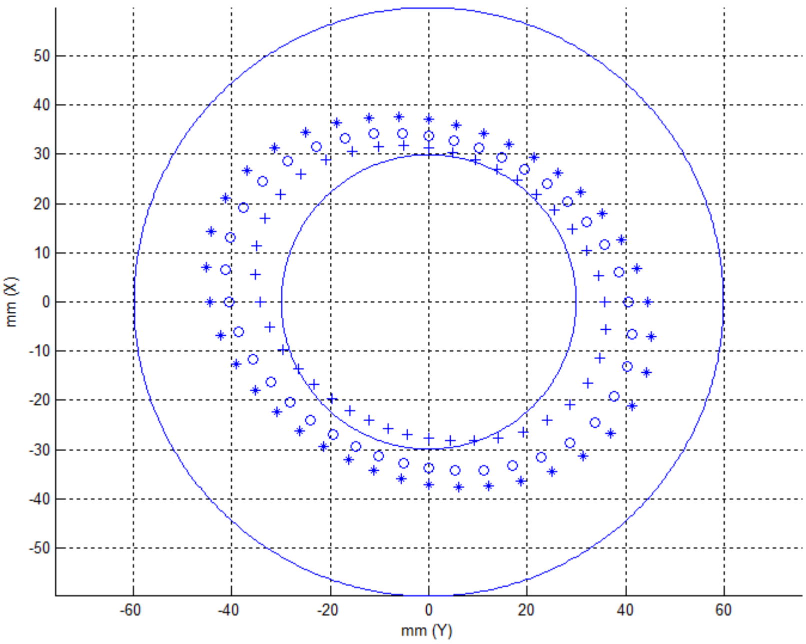

Fig. 6. TMDD and EMDD magnitudes for point C

The calculated TMDD magnitudes are greater than the displacements obtained from the empirical approaches. The TMDD magnitude is getting closer to EMDD magnitude when β=0.30.

6.2 Results for GPS Network

The up component was not considered in the GPS network processing. The results were calculated for two dimensional network (n = north, e = east). The components of the displacement circle were obtained for point OBC1, OBC2 and OBC3 which are called object points and the radius was computed as r=7.1 mm. Then, the EMDD magnitudes were also calculated, using the 2nd Empirical Technique: Step Approach Testing.

To compare the magnitude with theoretical approach the minimum detectable displacements were computed at 40 different directions from Eqs. (19) - (23) with α = 0.05 and β = 0.20. The displacements magnitudes were obtained for α = 0.05 and β = 0.30 as well (Fig. 7, Fig 8, Fig. 9).

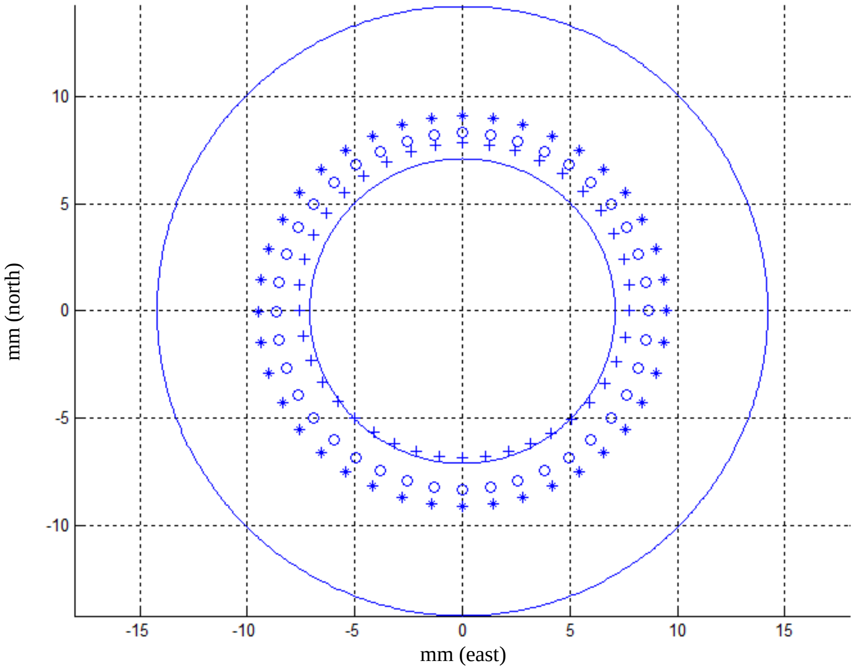

Fig. 7. TMDD and EMDD magnitudes for point OBC1

The field between two circles at Fig. 7, Fig. 8 and Fig. 9 are corresponds to radius (r, 2r) interval that were obtained from 1st Empirical Technique: Using Displacement Ellipse shown as sect. 3.1. This algorithm was used for calculating the radius of circles.

"+" refers to displacement magnitude that were obtained by the 2nd Empirical Technique: Step Approach Testing whose minimum detectable displacements magnitudes of each points were calculated and shown for at 10 grade intervals.

“*” and “°” shows the TMDD magnitudes for α=0.05 and β=0.20, α=0.05 and β=0.30, respectively.

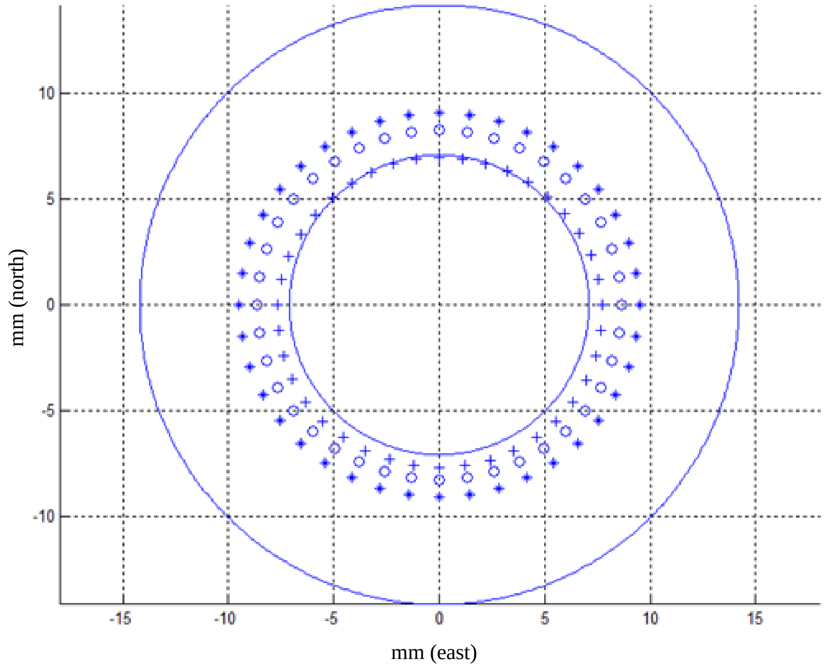

Fig. 8. TMDD and EMDD magnitudes for point OBC2

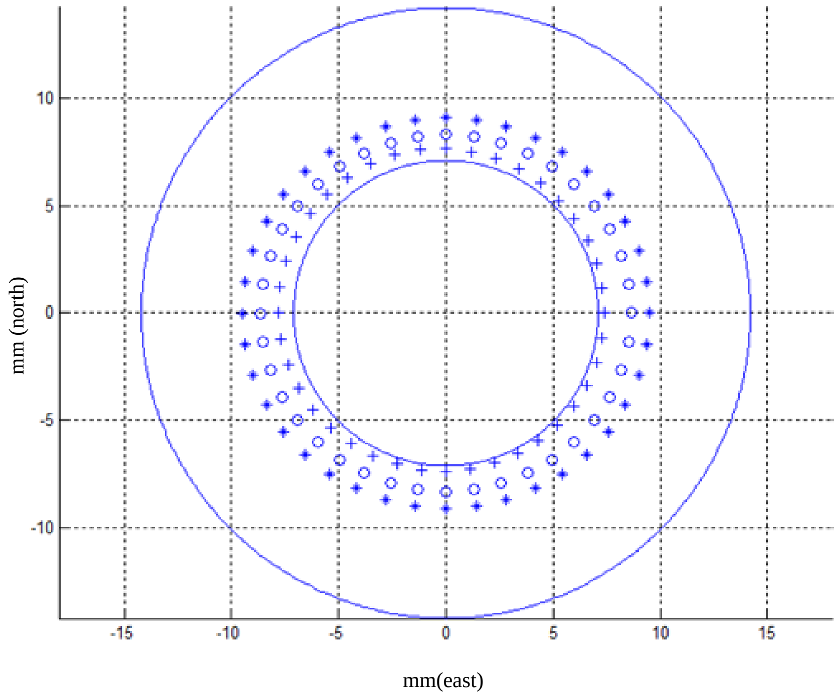

Fig. 9. TMDD and EMDD magnitudes for point OBC3

The calculated TMDD magnitudes are greater than the displacements obtained from the empirical approaches. The TMDD magnitude is getting closer to EMDD magnitude when β=0.30 for GPS network.

7. CONCLUSIONS

The MDD is very important for design of the deformation monitoring network. This value classically are estimated based on the $\alpha = 0.05$ and $\beta = 0.20$. However, since it is a theoretical value, magnitude of TMDD changes depending on the α and β. In this paper, two different EMDD techniques have been shown for estimating the MDD. The magnitudes of the TMDD techniques are greater than the ones of the EMDD techniques. Accordingly, the simulation of the displacement should be based on EMDD techniques for the reliability estimation of the deformation analysis method. Otherwise, obtained result values from TMDD can be rather optimistic than most probable values.

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