The Russian State Geodetic Coordinate System — 2011, Founded on Satellite Network

Victor POPADYEV, Igor STOLYAROV, Vladimir GOROBETS, Russia

Work name: Igor Stoliarov, Head of Department, Viktor Popadev, Haed of Division and Vladimir Gorobec, Head of Division (Russia)

Key words: Coordinate System, Geodetic Network, Geocentric Ellipsoid, Astrogeodetic Network

Are given an overview of the state geodetic coordinate system 2011 and the main results of its implementation in the territory of the Russian Federation. The parameters of the transition from the coordinate system used on the territory of the Russian Federation at present, by 2011, the system of coordinates. The prospects of further development of the system of coordinates for the period 2011 to 2020.

Даны общие сведения о государственной геодезической системе координат 2011 года и основные результаты ее практической реализации на территории Российской Федерации. Приведены параметры перехода от систем координат, используемых на территории Российской Федерации в настоящее время, к системе координат 2011 года. Показаны перспективы дальнейшего развития системы координат 2011 года на период до 2020 года.

The Russian State Geodetic Coordinate System — 2011, Founded on Satellite Network

Victor POPADYEV, Igor STOLYAROV, Vladimir GOROBETS, Russia

INTRODUCTION

Center of geodesy and cartography (formerly TsNIIGAiK) is known by own fundamental works on Earth's figure theory (Molodensky M. S., Yeremeev V. F., Yurkina M. I.) and theoretical geodesy (Pellinen L. P.). In 1940 prof. A. A. Isotov was determined the ellipsoid of reference with the semimajor axe 6 378 245 m and flattening 1/298,3, which was the last ellipsoid, determined by astro-geodetic methods, namely "Krassowsky ellipsoid". The surface of this ellipsoid was used as reference in USSR in coordinate systems of 1942 and 1995. In 1962 prof. L. P. Pellinen was determined the new parameters of mean Earth's ellipsoid with semimajor axe, familiar by each geodesist: a = 6 378 137 m; α = 1 : 298,250. From 1962 until 1980 only the error of semimajor axe's value was decreased from 50 to 2 meters.

The development of the new geocentrical coordinate systems GSK-2011 was performed within 2000—2012 years.

The orientation of axes in GSK-2011 corresponds to ones in ITRS, recommended by International Earth Rotation and Reference Systems Service (IERS), and International Association of Geodesy (IAG): origin located in Earth's mass centre; Z-axis directed towards CIO, and zero meridian plane is parallel to astronomical one. The universal gravity constant, light speed and 1 second are determined the same as in CODATA.

The Normal Earth's parameters are determined by the Pizzetti's theory of level ellipsoid of rotation. The surface of this ellipsoid used as reference in geometrical applications and its gravity field as normal field in gravimetrical studies.

The fundamental geodetic parameters of GSK-2011 consists of four constants:

—geocentrical gravity constant (incl. mass of atmosphere) $GM = 398\ 600{,}4415 \cdot 10^9$ m^3/s^2 (in units of Earth's dynamical time),

— mean angular velocity of Earth's rotation $\omega = 7{,}292115 \cdot 10^{-5}$ rad/s,

— semimajor axe of ellipsoid (equatorial radius of Earth) a = 6 378 136,5 m,

— flattening (compression) of ellipsoid α = 1:298,2564151.

It is supposed that fundamental constants must have natural values and be determined without any hypothesis, then we have to establish ω, GM, normal potential on the ellipsoid U_0 and dynamical coefficient J_2. Since the mean Earth's ellipsoid represents the Earth's global characteristics, U_0 must be equal to the mean value of real potential on the sea level W_0 (not depends of tides), but the semimajor axe is preferred. By the same way flattening α of ellipsoid used as fundamental instead of J_2.

The mean angular velocity of Earth's rotation ω is established by the IERS, the secular deceleration $d\omega/dt = -4{,}5\cdot 10^{-22}$ rad/s^2 corresponds to the observed increase of day's duration by last 2700 years.

1.1. Other geometrical constants

semiminor axe $b = a(1-\alpha) = 6\ 356\ 751,\ 75795\ 56$ m,
first eccentricity squared $e^2 = 2\alpha - \alpha^2 = 0,\ 00669\ 43981\ 05662\ 14$,
second eccentricity squared $e'^2 = 0{,}00673\ 95151\ 02799\ 4$.

1.2. Other physical constants

1.2.1. Normal potential of level ellipsoid

The closed expression of the gravity potential V^e of the level ellipsoid with axes *a, b* is determined in spheroidal system *u, v, w* [*Molodensky et al. 1962*, (II.28)]. On the surface of the ellipsoid GSK-2011 the normal potential is

$$U_0 = V^e{}_{H=0} = \tfrac{GM}{c}\,\mathrm{arcctg}\,\tfrac{b}{c} + \tfrac{\omega^2 a^2}{3} = 62\,636\,856,750633 \text{ m/s}^2.$$

This value U_0 corresponds to the mean W_0 on the sea level and differs from its[1] $(U_0 - W_0)/W_0 \sim 10^{-9}\ldots10^{-8}$: with the residual difference 0,2. . . 0,5 m^2/s^2, that at the modern level of accuracy permits to neglect systematic component in gravimetric quasigeoid height $\Delta\zeta = (U_0 - W_0)/\gamma \sim 5$ cm. In systems WGS-84 and ITRF still uses a system of fundamental constants GRS-80 with small changes, adopted by XVII General assemble of IUGG. Since 1990th the discrepancies of the value of U_0 is noticeable[2], derived from a heterogeneous co-processing of space geodesy data (satellite laser ranging, satellite altimetry) and terrestrial gravimetry. Nevertheless IAG recommended to maintain the GRS-80, noting its possible modification in the future. In physical parameters of the coordinate system GSK-2011 this modification is made.

The second zonal coefficient (non-normalized) of the normal potential:

$$J_2 = e^2/3 - {}^2/_{45}\, me^3/q_0 = 0{,}00108\ 26361\ 425398.$$

The relative change in the dynamic coefficient is $\dot{J}_2 / J_2 = -2{,}7774\cdot 10^{-6}$. The difference of the calculated J_4 from the real one is about 50%, but the establishment of the normal field by the first few harmonics *N* to of the geopotential (for example, as Clairaut theory) is impractical (to avoid of considering of the height of the surface level above the reference one). The inclusion of permanent tide is also inappropriate.

For purposes to process the levelling and gravity survey use of Helmert formula 1901—1909 can be possible, since the normal gravity formula is very similar:

[1] *Бурша М. и др.* Оценка точности геопотенциальных моделей EGM X01—X05, EGM 96 M. Геод. и карт., 8, 1998, с. 10—13.

[2] *Бурша М.* Фундаментальные геодезические постоянные. Геод. и карт., 5, 1996, с. 15—22. *Burša M., Kenyon S., Kouba J., Šima Z., Vatrt V., Vítek V., Vojtíšková M.* The geopotential value W_0 for specifying the reativistic atomic time scale and a global vertical reference system. J. of Geodesy, (2007), 81: 103—110. ($W_0 = 62636856{,}0 \pm 0{,}5$ м^2/с^2.)

$$\gamma_0 = 978\ 032{,}69634(1{+}0{,}00530\ 243_0 \sin^2 B{—}0{,}00000\ 58 \sin^2 2B$$
$$—0{,}00000\ 0032 \sin^2 B \sin^2 2B) \text{ mGal.}$$

2.1. Normal heights changes

The normal heights do not depend on the coordinate system and depend weakly on the parameters of the Normal Earth, since the change in these parameters affects in the normal gravity formula and its vertical derivative. Increment of the γ^m by $\Delta\gamma^m$ = 1, 2, 5, 10 mGal on the heights 500 m, 1, 2 and 5 km changes the normal height value by $\Delta H^\gamma = H^\gamma\ \Delta\gamma^m / \gamma^m$:

$\Delta\gamma^m$	H^γ =500 m	H^γ =1 km	H^γ =2 km	H^γ =5 km
1 mGal	0,5 mm	1 mm	2 mm	5 mm
2 mGal	1 mm	2 mm	4 mm	10 mm
4 mGal	2 mm	4 mm	8 mm	20 mm

Changing the equatorial constant γ_e fully enters to the new value γ^m. The actual technique of processing of precise levelling based on the Helmert normal gravity formula (1901—1909)

$$\gamma_0 = 978030(1 + 0{,}005\ 302 \sin^2 B — 0{,}000\ 007 \sin^2 2B) \text{ mGal.}$$

The corresponding vertical gradient derived by Helmert varies from –0.3088 mGal/m at the equator to –0.3083 mGal/m on a pole. Differences of γ^m at different latitudes due to changes γ_0 and $\partial\gamma/\partial H$ gradient according to Helmert and GSK-2011, see Table. 1.

Table. 1.

B	$\Delta\gamma_0$, mGal	$\Delta(\partial\gamma/\partial H)$, mGal/m	$\Delta\gamma^m$, mGal
0°	2,6963	$-6{,}4318\cdot10^{-7}$	2,6963
10	2,841	$-7{,}1767\cdot10^{-7}$	2,841
20	3,2119	$-9{,}5461\cdot10^{-7}$	3,2119
30	3,6486	$-1{,}3827\cdot10^{-6}$	3,6486
40	3,9668	$-2{,}0164\cdot10^{-6}$	3,9668
50	4,0423	$-2{,}8239\cdot10^{-6}$	4,0423
60	3,866	$-3{,}7092\cdot10^{-6}$	3,866
70	3,545	$-4{,}5227\cdot10^{-6}$	3,545
80	3,2496	$-5{,}0986\cdot10^{-6}$	3,2496
90	3,1311	$-5{,}307\cdot10^{-6}$	3,1311

The corresponding changes of the normal height on the territory of Russia at an altitude of 5 km will be less than 2 cm. For the average land elevation above sea level (230 m) changes in normal height will be less than 1 mm. Redrawing of the maps is not required, if necessary, the corrections on the benchmarks are easy to calculate.

2.2. Historical review

In Russia, there are several generations of coordinate systems.

1. Isolated triangulations. The Laplace points used as a support, which can be determined by various astronomical observatories. Longitude observations limited by accuracy of chronometer. The processing is performed by the method of “deployment” to the surface of the reference ellipsoid, adjustment is performed on the ellipsoid, from the 1930th. the Gauss-Kruger projection

adopted. The last such coordinate system was a system of 1932, processed and adjusted on the Bessel ellipsoid.

2. Unified triangulation network. For network orientation the Laplace points used, which defined from the Pulkovo Observatory, longitudinal determination was performed using accurate time signals, in the processing of network used the method of “projection” on the surface of ellipsoid (Krassowsky, 1940). Since established the coordinate system of 1942. Changing size of reference ellipsoid led to critical changes in the plane coordinates, particularly the north component (hundreds of meters), and the relative stability of geodetic curvilinear coordinates.

3. Triangulation network and traverse based on points, determined by the methods of space geodesy: Space Geodetic Network (military), Doppler Geodetic Network. Coordinates of points Space and Doppler Geodetic Network obtained initially in the geocentric system, shifted to the reference ellipsoid Krassowsky. When processing is applied method of “projection”, the adjustment is made on the surface of the ellipsoid Krasovsky. Thus, a system of coordinates of 1995 is formally geocentrical, but manually shifted. Preserved size and orientation of the reference ellipsoid saved the geodetic branch of critical changes of geodetic coordinates and time-consuming re-release of topographic maps that have existed only in analog form.

4. The same network triangulation and traverse based on points, determined by a GNSS: Fundamental Astro-Geodetic Network (about 50), Precise Geodetic Network (about 300, see Fig. 1) and Satellite Geodetic Network (about 4000), the coordinates of which are given in the geocentric system and reduced to the common terrestrial ellipsoid GSK-2011. When processing is applied the same method of “projection”, the adjustment is made on the surface of the ellipsoid GSK-2011. Thus obtained the coordinates of about 300 000 geodetic points in coordinate system of 2011. Changing of reference surface led to changes in geodetic coordinates (see. Fig. 2) and the critical changes of plane rectangular coordinates (see. Fig. 3) of the geodetic points and terrain objects, but since most of the topographic maps is translated into electronic form, their conversion to GSK-2011 is performed in semiautomatic mode.

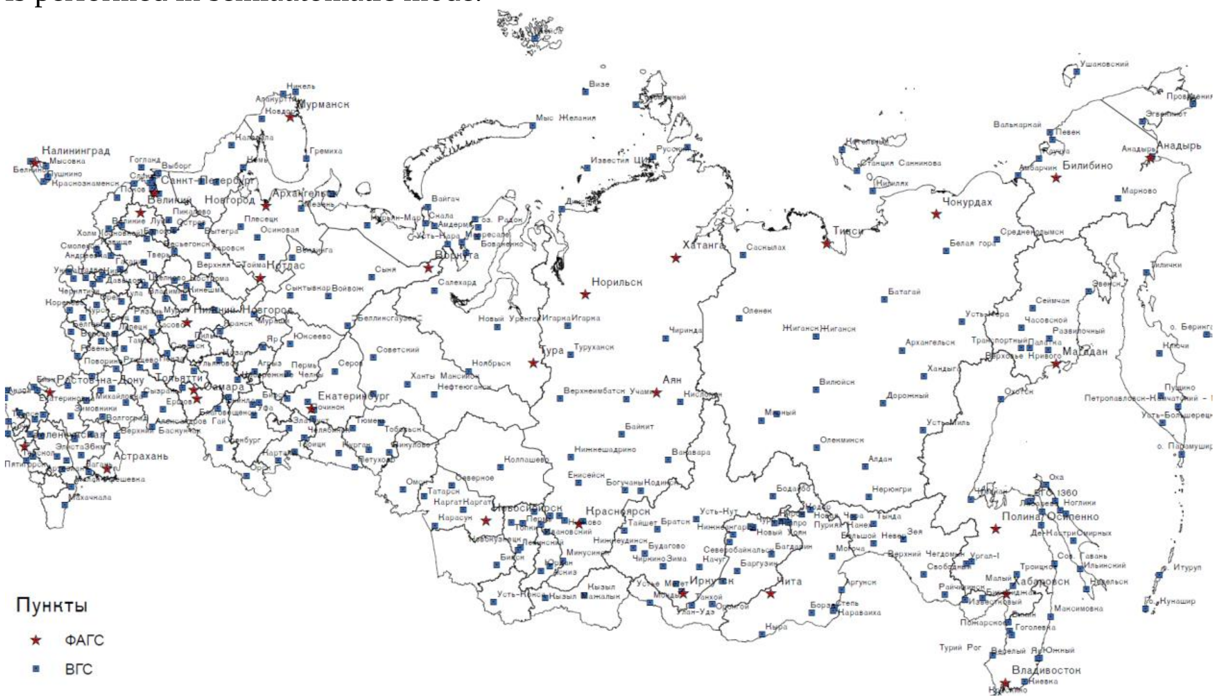

Fig. 1

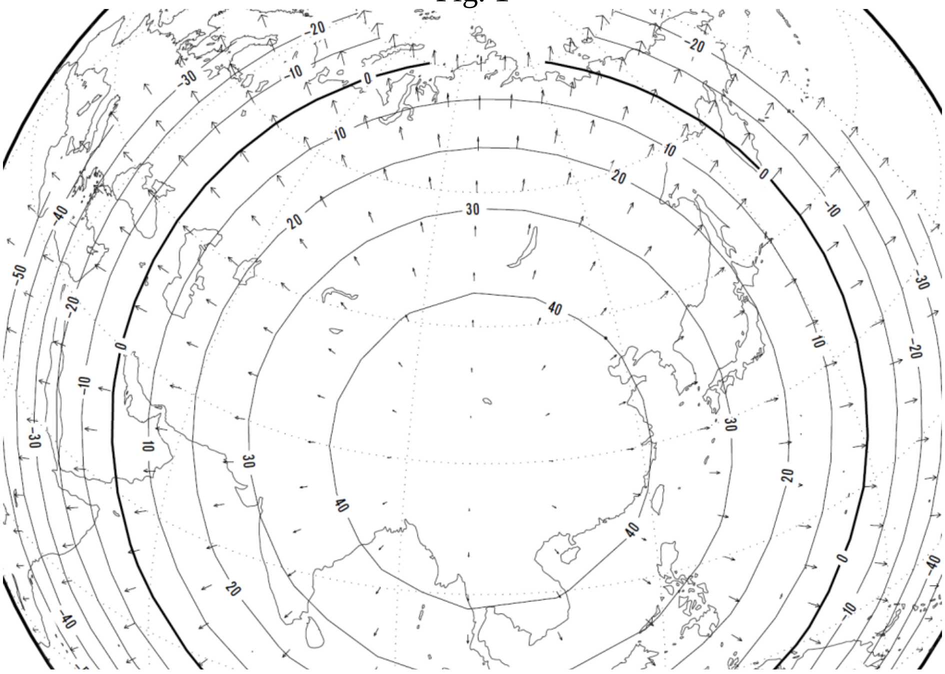

Fig. 2

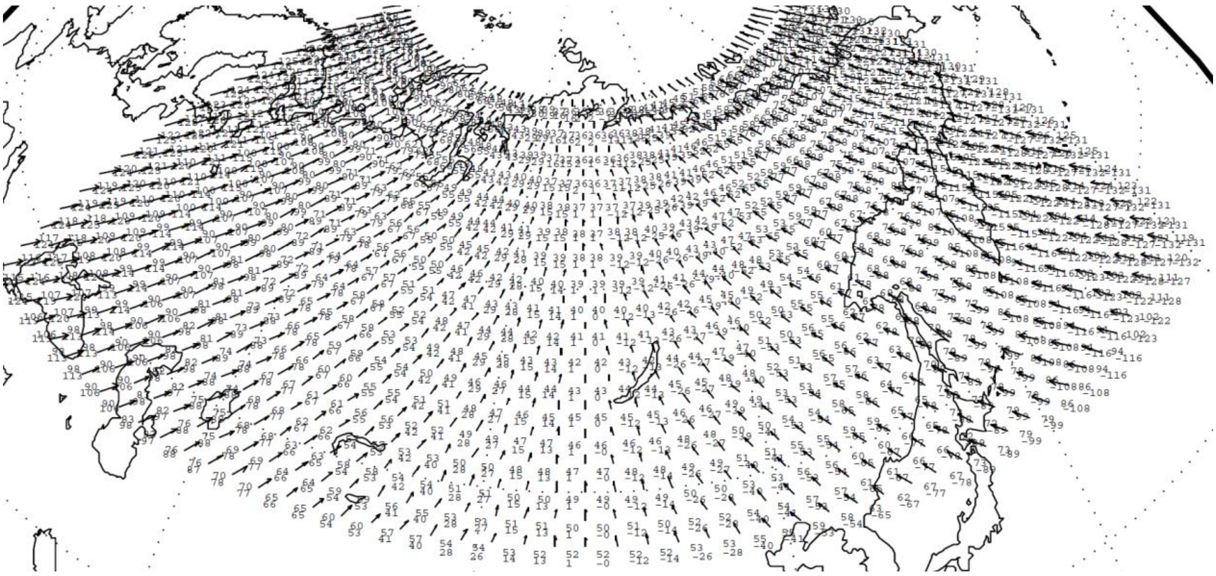

Fig. 3

2.3. The formation of GSK-2011

2.3.1. Adjustment of FAGN and satellites networks in a geocentric coordinate system

The basis of the GSK-2011 is a satellite geodetic network, which includes the fundamental astronomical and geodetic network (FAGN); Precision Geodetic Network and Satellite Geodetic Network.

The spatial position of FAGN points determined by the GNSS-methods in common terrestrial coordinate system relative to the center of mass of the Earth with an error by order 10…15 cm, and the error of the relative position of any points FAGN does not exceed 1…2 cm at the plane position and 2…3 cm considering their changes over time.

The coordinates of FAGS points on epoch 2011.0 were determined by the network adjustment based on the IGS points. Foreign IGS items used in order to make the network FAGS rigid and reliable as the number and geographical distribution of the IGS points in the territory of Russia is not optimal.

Mean-square error of the adjusted coordinates FAGN points is 1…10 mm in plane (Vladivostok 1.9 cm) and 0.2…1.5 cm in height (Vladivostok 2.14 cm). At 1 January 2016 FAGN consists of 58 points, 48 of which are permanent and 10 are periodically determined. The structure consists of 13 FAGN points of Russian Academy of science, 5 points of RosStandard and 40 points RosRegistry, 3 points are combined with the VLBI, 8 points combined with differential correction points.

2.3.2. <u>Adjustment of astro-geodetic network on the ellipsoid GSC-2011</u>

When passing to the surface of an ellipsoid, in the measured directions have been introduced corrections:

— for plumb-line deflections;

— for the transition from the normal section of the ellipsoid to the geodetic line;

— the height of the observation point above the surface of ellipsoid;

In the measured azimuths has been introduced the same corrections, Laplace correction and correction for inaccuracy of geodetic coordinates used while transition from astronomical azimuth to geodetic.

The measured bases was also corrected:

— for putting to the horizon and the height of the observation point above the surface of ellipsoid;

— for plumb-line deflections;

Due to the fact that originally required the reference ellipsoid while adjusted European Astro-Geodetic Network, the data prepared in such a way that when it did not have sufficient or insurmountable challenges.

In this connection, to carry out the adjustment on an ellipsoid GSK-2011 in the adjustment software of the Centre for Geodesy and Cartography were made the small corrections:

— entered ellipsoid GSK-2011;

— the preliminary coordinates are calculated from ones of system 1995 by the 7-parameter transformation;

— components of plumb deflections above ellipsoid Krassowsky, recalculated on an ellipsoid GSK-20112011.

— introduced a parameter of adjustment in the GSK-2011;

— quasigeoid height calculated on a global model;

— a list of reference points is given in Gauss-Kruger projection in GSK-2011;

After Astro-Geodetic network adjustment on the basis FAGN, Precise and Satellite Network points (combined with 1—2 class points of Astro-Geodetic network) obtained all the coordinates of geodetic points, even those whose safety is unknown.

Geodetic network of 3—4 class does not form a continuous network, so common 1—4 class network adjustment inappropriate. Geodetic network of 3—4 class adjusts by insertion after the Astro-Geodetic network 1—2 class, which are used as a support for them. At this stage, as the reference points in addition used Precise and Satellite points, combined with 3—4 points. The coordinates of the network points 3—4 class can also be obtained from the deformation matrix (in areas with a dense satellite network).

In all variants of adjustment the internal network structure is not subjected to significant deformations. Local deformations are eliminated, as a rule, quite smoothly. This follows from the minor changes in the values of mean-square of unit weight error and the distribution of corrections in the measured elements. One reason for this is quite rare strongholds network. One of the most important results of the adjustment in the GSK-2011 is to provide a method to convert the coordinates of all the points from the coordinate systems of 1942, 1995 and local systems to the GSK-2011 while eliminating local deformations inherent in each of these coordinate systems. R. m. s. conversion error does not exceed 5 cm (max 15 cm). Such a high conversion accuracy due to the presence in the now relatively dense network of points FAGN, Precise and Satellite Network. They have been used in the adjustment of Astro-Geodetic network in the GSK-2011 as a support that has allowed to distribute coordinates in GSK-2011 to the all geodetic points. The main source of error is the assumption that the used angles and plumb line deflections not changed from the time of measurement.

2.3.3. Coordinate changes of geodetic points in GSK-2011

Vectors of coordinate changes are shown in Fig. 4.

2.3.4. Deformation matrix

Deformation matrix (correction grid) astro-geodetic network with respect to the satellite is created in the accumulation of satellite measurements on the old geodetic points of 1—2 class. From adjustment are obtained coordinates in the geocentric system (along the way, on the basis of pairs of coordinates of geodetic points in the two coordinate systems can be found relative local orientation elements). In old geodetic points of 1—2 class formed the difference ΔB, ΔL between the adjusted coordinates and catalog. If the orientation parameters are known adjusted coordinates can be translated into the reference system and a residual difference formed. The differences ΔB, ΔL interpolated on a regular grid with a step 1—2 arc minutes and form the regular grid. Such grids are created for the system 1942 and 1995 (see Figure 4.).

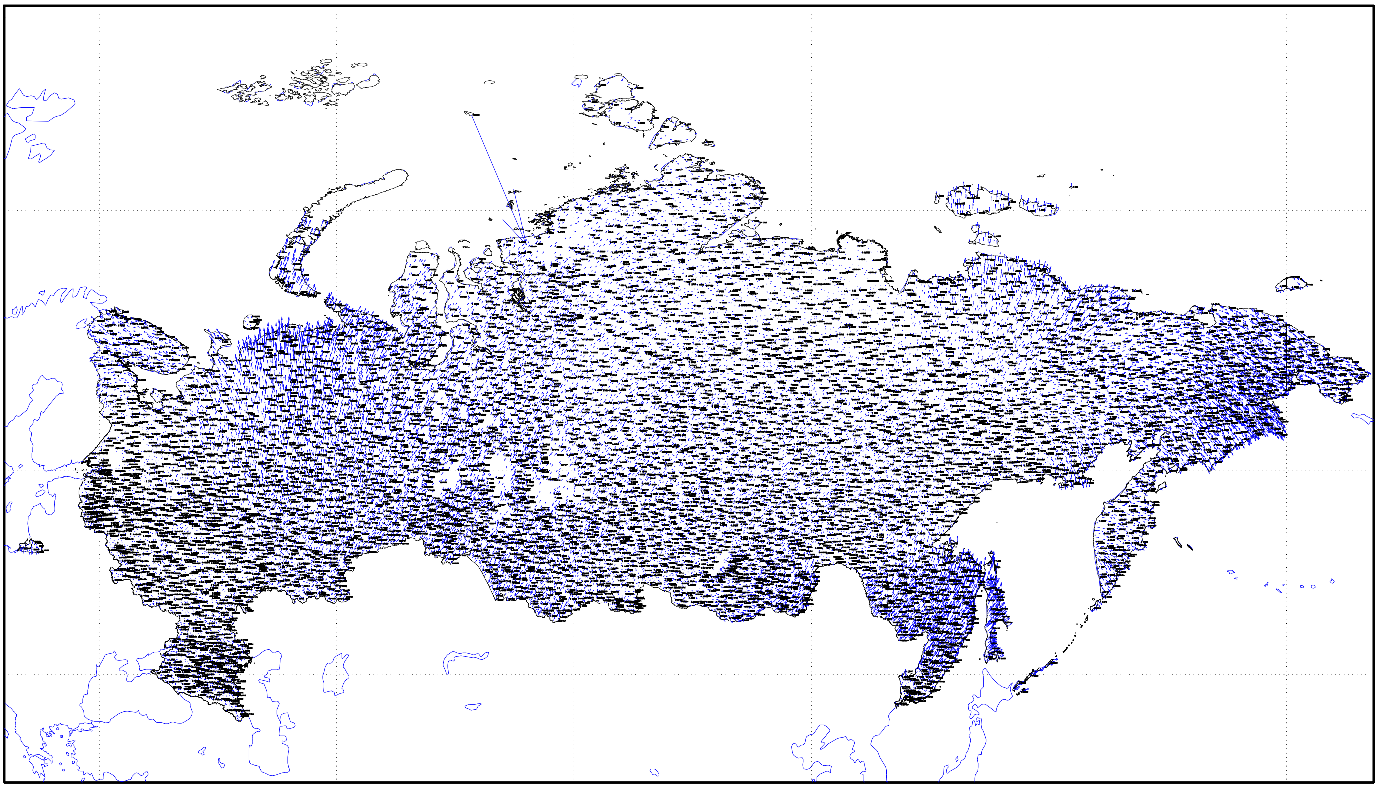

Fig. 4.

According to the coordinates *B*, *L* the nodes of deformation matrix are founded and current correction determined by bi-cubic interpolation. The correction introduced in the current geodetic coordinates, the result obtained in the geocentric coordinate system. This approach is justified, in particular, by the fact that the latest versions of RTCM protocol used in network RTK definitions provided for the transfer of the 7 parameters of the local projection and grid 4×4 of residual corrections for subsequent bi-cubic interpolation of a point location of the user. The algorithm of calculation of the seven parameters should be stored in the network differential station

According to the control points are obtained residual difference and conversion error can be estimated. In areas with a dense network of existing satellite points r. m. s. of the coordinate transformation is about 5—10 cm. In regions with sparse satellite network, as well as in the northern regions error is more than 10 cm.

BIBLIOGRAPHY

1. Molodensky M.S., Eremeev V.F., Yurkina M.I. (1962) Methods for study of the external gravitational field and figure of the earth.

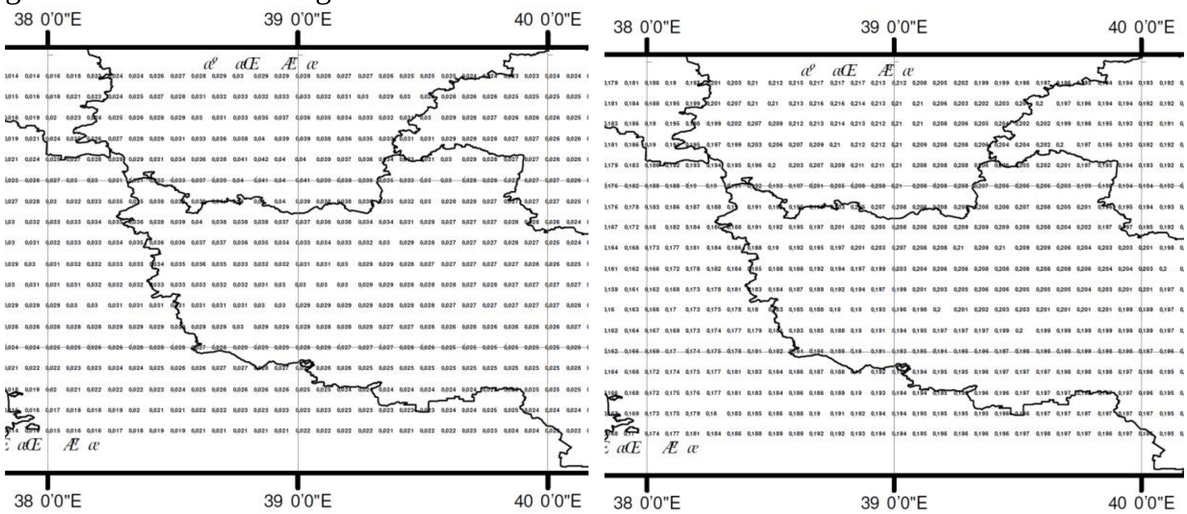

Fig. 5. Elements of deformation matrix: ΔB (left) and ΔL (right) in arc seconds