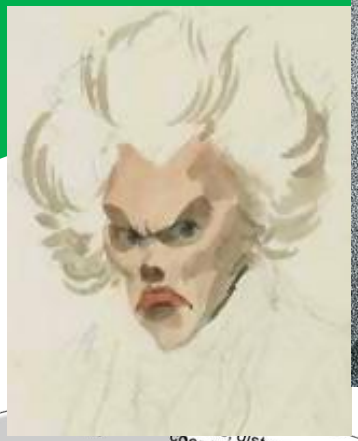
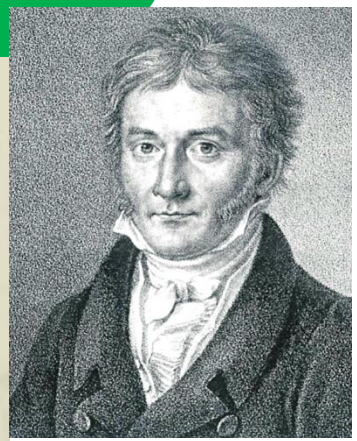
25 years of teaching least squares adjustment

by
Ulla KALLIO, FGI, Finland

Least squares method since 1804 or 1794

It is an old method, we need something more popular





$$v^T P v = \min$$

$$\Rightarrow \quad (Ax - y)^T P(A$$

$$\Rightarrow \quad (x^T A^T P - y^T P)($$

$$\Rightarrow \quad x^T A^T PAx - x^T A^T Py - y^T PAx + y^T Py =$$

$$\Rightarrow \quad 2x^T A^T \quad PA - y^T PA = 0$$

$$\Rightarrow \quad A^T PA - 2y^T PA = 0$$

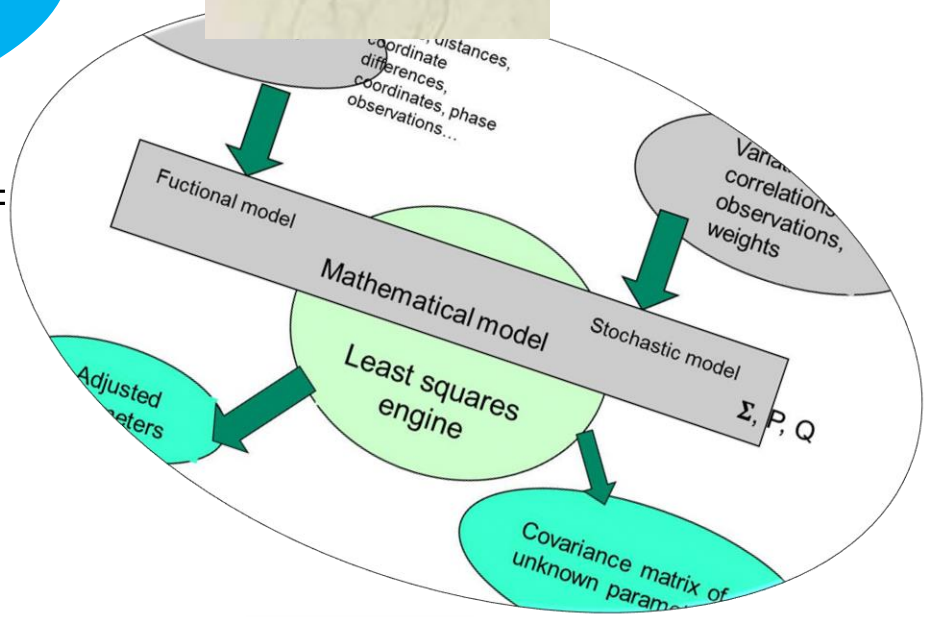
$$\Rightarrow \quad x^T A^T PA = y^T$$

$$\Rightarrow \quad A^T PAx = A^T Py$$

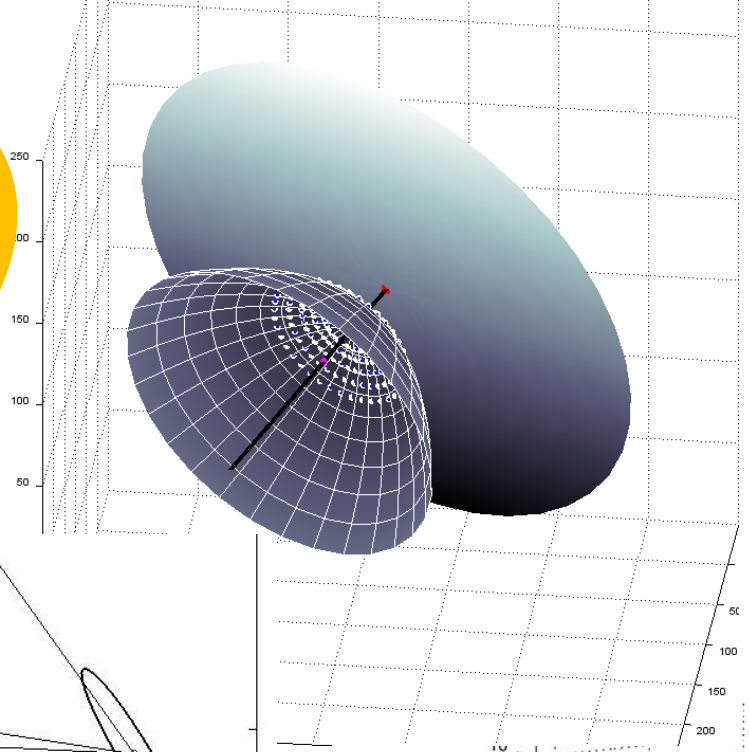
Isn't it already programmed in the applications ?

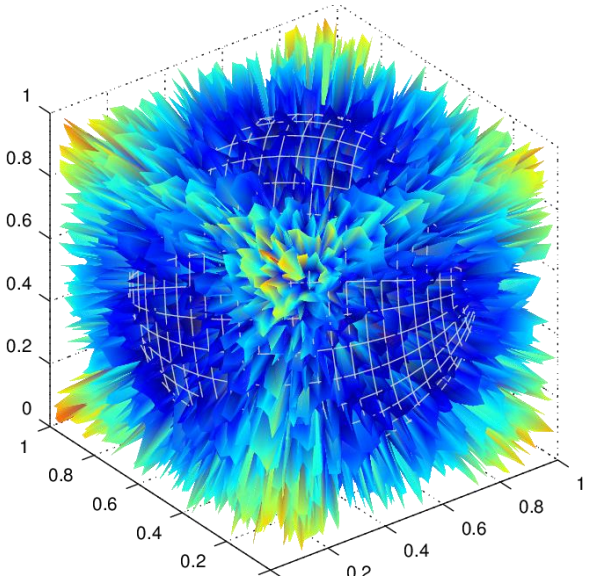
Difficult subject

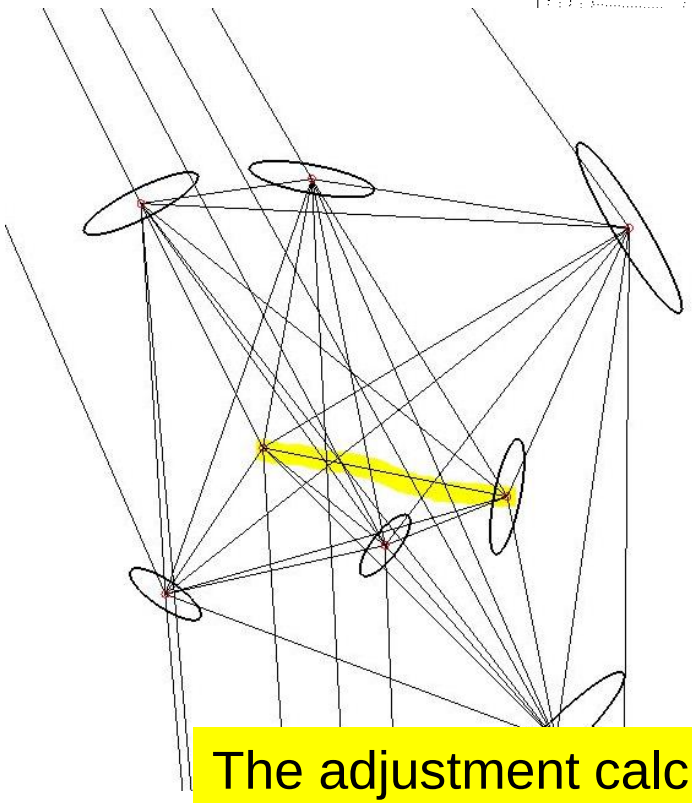
Anybody still need it?

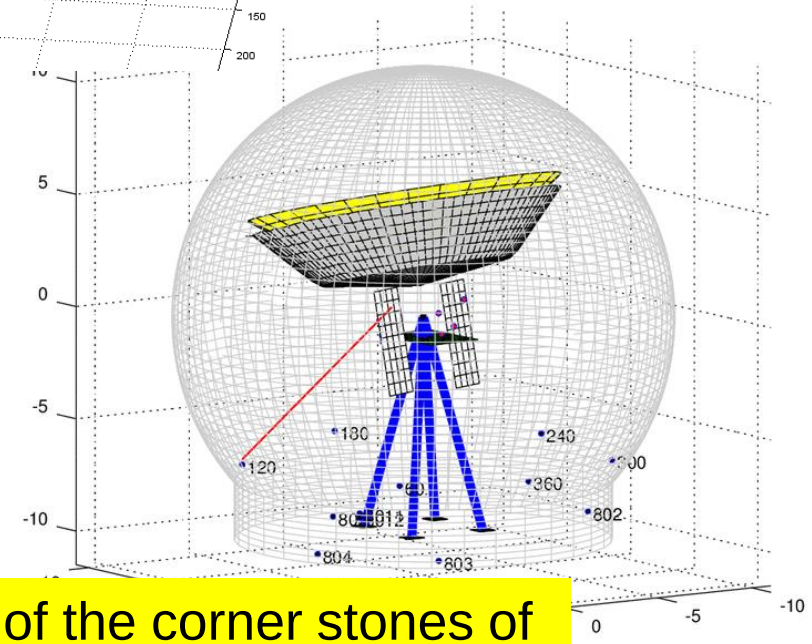


Anybody still need it?

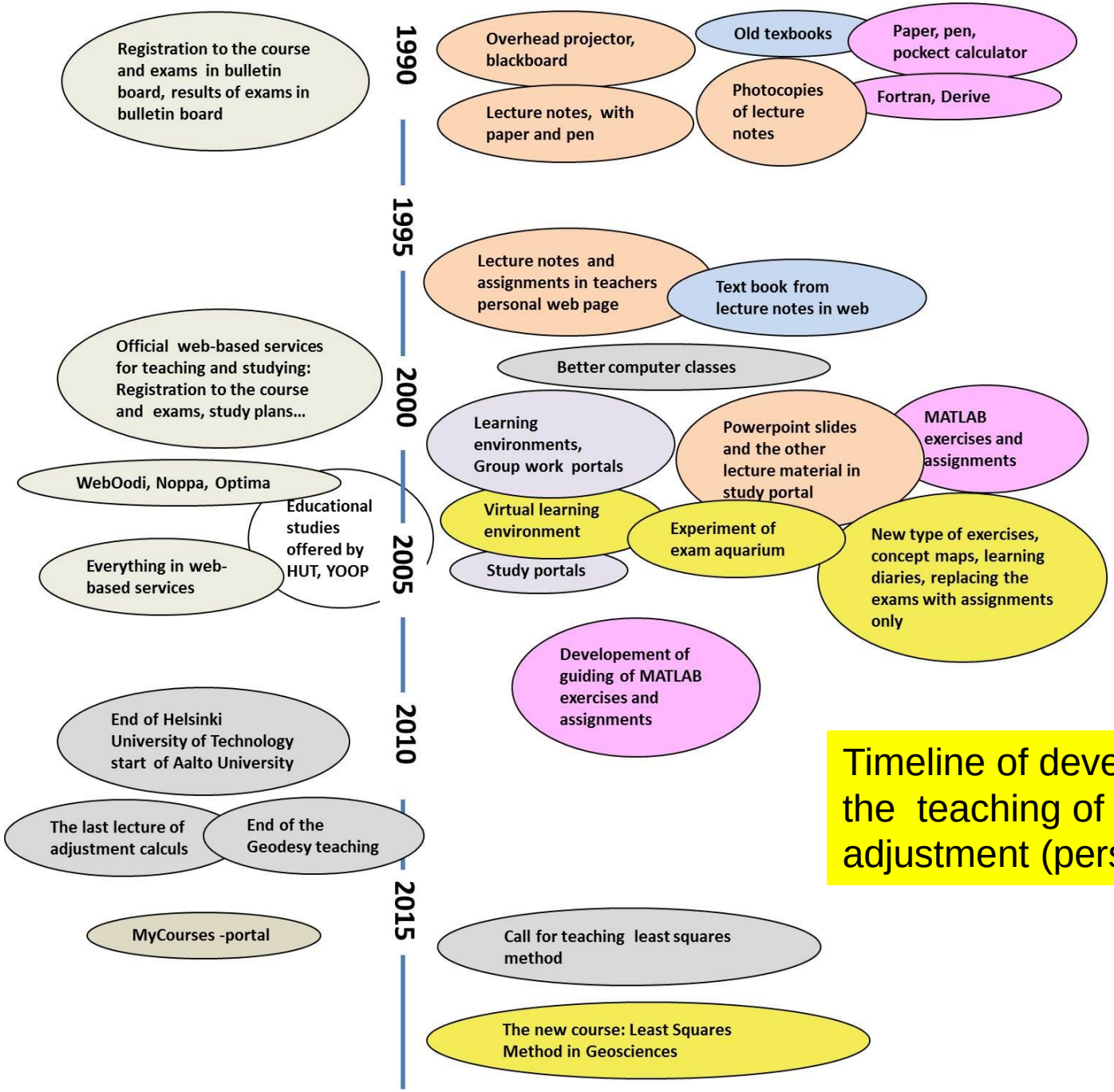








The adjustment calculus is one of the corner stones of the professional knowledge of geodetic surveyors.



Timeline of developments in the teaching of least squares adjustment (personal view)

1990

ILMOITTAUTUMINEN OPINTOJAKSOLLE

Koodi Maa-6.202 nimi Tasoituslasku

Opintojakso alkaa

Ilmoitus jätetty ____________, poistetaan

ilmoittauduttava opintojakson pitopaikassa op

Huom!

TÄYTÄ PAINOKIRJAIMIN!

Op.k.-n:o	Nimi	Koulutus-ohjelma	Vsk	Op.k.-n:o
37425W	Anna Hyvönen	3	III	
37491L	EIJA ORAVA	1	III	X
30619H	Petri Takala	1	N	
36165K	Jyrki Mononen	1	III	X
36157A	Hannu Koivula	1	III	X
36159C	Jarkko Koskinen	1	III	X
34927F	MARKUS TÖRMÄ	1	IV	X
36146K	Mari Heinonen	1	IV	
36167M	Katri Oksanen	2	IV	
33731K	Paula Takala	2	VI	
37446D	MIKA MANNINEN	2	III	X
37447N	MIKA MANNERVESI	2	III	X
33748	MATTI HAVUKKALA	1	N	

TEKNILLINEN KORKEAKOULU
Geodesian ja kartografian laboratorio

ILMOITTAUTUMINEN OPINTOJAKSOLLE

Koodi MAA - 6.202

Nimi TASOITUSLASKU Op. Lab.ins. Ulla Lankinen

Opintojakso alkaa Lukujärjestyksen mukaan

Ilmoitus jätetty 8.1.1992

Ilmoitus poistetaan 1. luennon yhteydessä

jonka jälkeen on ilmoittauduttava opintojakson pitopaikassa opettajalle.

Huom! Tarkemmat tiedot: vrt. opinto-opas + opetusohjelam

TÄYTÄ PAINOKIRJAIMIN

Opintok. n:o	Nimi	Koulutusohjel-ma	Vsk	Opintok. n:o	Nimi	Koulutusoh-jelma	Vsk
40280U	Susanna Vesimäki	M	II	38818B	Pasi Lappalainen	M	II
37425W	Anna Hyvönen	M	4	38809N	VIRPI SARVIMÄKI	M	III
40179L	Outi Koskenniemi	M	II	38812S	PAULIINA LEINONEN	M	III
33768F	Markus Seppänen	MN	IV	36168N	Hanna Peltonen	M	
36147L	Kaukonen Kirsi	M	5	38802E	Tord Lindberg	M	II
38844K	Jari Salila	M	3	40221V	Kimmo Virtanen	M	II
38819C	Tuomas Lehtonen	M	3	36836J	Ilkka Suurkuukka	M	II
38836A	Virpi Pennanen	M	3	36154U	Samuel Kivinenta	M	5
40194K	MARKO HANNONEN	M	II	40179R	MARKKU HUOTARI	M	II
38830R	TOMMI NORRI	M	II	38806K	RISTO ILVES	M	II
40188P	Juha Helminen	M	II	[illegible]	[illegible]	M	II
[illegible]	[illegible]	M	II	37427U	Hanna Heikkilä	M	IV
38810P	SATU KANKAALA	M	3	37413P	Anu Rinninen	M	IV
37441N	Ilkka Laakso	M	III	40187C	Eero Karjalainen	M	II
41426F	JAAKKO WSITALO	M	I	37440M	Päivi Korpela	M	4
36681N	Ritmo Salonen	M	II	40180S	HANNU HUTTUNEN	M	II
36138A	Palomiemi Sari	M	N	40201U	Anne Nieminen	M	II
38829R	Jari Mäkinen	M	II	37426T	Elina Erkkilä	M	4
40217R	Maarit Tuokas	M	II	41186B	Soile Kankkunen	M	II
36465E	Markku Lindgren	M	II	38799B	Tore Grönlund	M	3
38815L	PEKKA SARKOLA	M	II	37460U	Pekka Pohto	M	4
31375H	Timo Mäkelä	M	N	40243L	Kari Jääminen	M	II
40FFFP	Paula Mäkelä	M	II	40193S	AULI MANNINEN	M	II
40222W	MARJUT VILKAINEN	M	II	38822F	JYRKI LÄMSÄ	M	II

Registration in bulletin board 1990 and 1992

$(A^TPA)^{-1}$ $\left(\frac{mm^2}{gon^2}\right)$

$$\begin{pmatrix} 8487,401787601971 & 1444,829511646691 & -873,937705372972 \\ 1444,829511646681 & 4932,901606715474 & 2,089749091251156 \\ -873,937705372972 & 2,089749091251155 & 7157,773437593957 \end{pmatrix}$$

Likiarvojen korjaukset

$$(A^TPA)^{-1}A^TPy = \begin{pmatrix} 0,001694423516724430 \\ 8,924553756535500 \cdot 10^{-4} \\ 0,022159494101628 8 \end{pmatrix} mm$$

Uudet likiarvot

$X_{73} = -223,289305576$
$y_{73} = -5853,751107755$
$Z_{73} = 1007,639159469$

1 page from 15 pages adjustment report of the student exercise in 1991

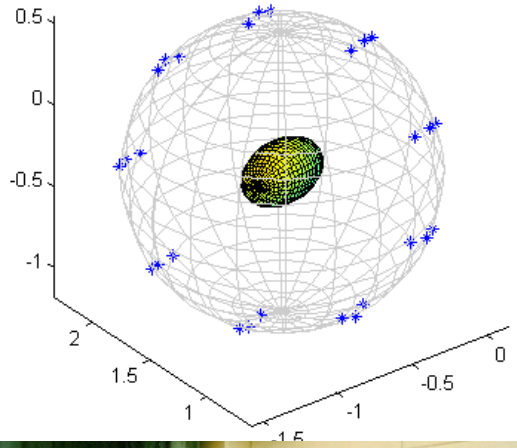
Uusi rakennematriisi $\left(\frac{gon}{mm}\right)$

$$A = \begin{pmatrix} -1,085961489411 \cdot 10^{-2} & 4,142362433881 \cdot 10^{-4} & 0 \\ 2,248777598832 \cdot 10^{-3} & -1,457082485900 \cdot 10^{-2} & 0 \\ 5,607530912731 \cdot 10^{-5} & 1,470069970721 \cdot 10^{-3} & -1,066457420891 \cdot 10^{-2} \\ -2,455626756211 \cdot 10^{-3} & -3,789870881761 \cdot 10^{-4} & -1,431196644861 \cdot 10^{-2} \end{pmatrix}$$

Uusi y-vektori

$$y = \begin{pmatrix} 2,12731 \cdot 10^{-5} \\ 1,341141081 \cdot 10^{-5} \\ 2,396251 \cdot 10^{-4} \\ -1,785561 \cdot 10^{-4} \end{pmatrix} gon$$

An example of the student work: ”fitting the sphere” programmed with MATLAB in 2014



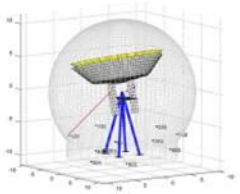
The first micro computer class room in the department of geodesy and cartography in 1986 with floppy disk operating systemPCs

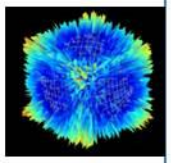
Benjamin Bloom, Max Englehart, Edward Furst, Walter Hill, and David Krathwohl published the *Taxonomy of Educational Objectives* in 1956. They categorize the cognitive levels or goals of learning. The six original categories are**:**

- **I Knowledge,**
- **II Comprehension,**
- **III Application,**
- **IV Analysis,**
- **V Synthesis, and**
- **VI Evaluation**.

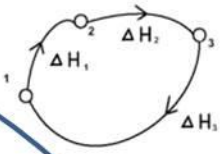
Bloom's taxonomy wheel in the learning of least squares adjustment

Create something new, use LSQ in new situation





Undertand LSQ adjustment as a part of the measurement process



$$v^T Pv = \min$$

I Knowledge

II Comprehension

III Application

IV Analysis

V Synthesis

VI Evaluation

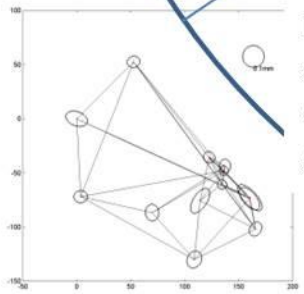
Understand the general solution of the inverse proble with LSQ

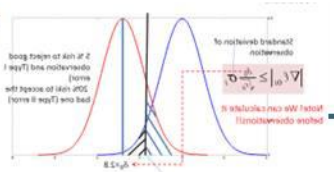
$$A^T PAx = A^T Py$$

Understand models with observation equations, condition equations and mixed model

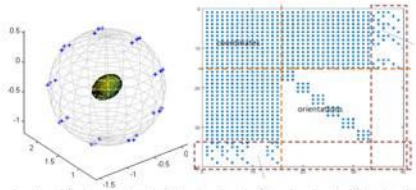
Students are able to program adjustment software package
Can enlarge the three basic adjustment models to sequential adjustment, Kalman filtering etc.
Can read scientific papers concerning least squares or related methods critically

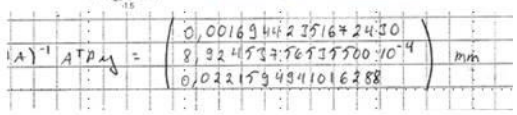
Can analyse the results of adjustment and the quality of measurements and the adjustment.
Can use the outlier detection and basics of testing theory





Student can apply the LSQ to typical surveying problems: Levelling networks, GPS-networks, transformation , tachymetric networks, fitting problems





Conclutions

- Learning is very much personal process, I believe learning by doing
- There is no guarantee that learning process of human being follows the Bloom's taxonomy steps
- Bloom's taxonomy is still a good tool for planning the teaching
- Teaching the least squares method is still important