Determining Topological Relationships Between 3D Legal Boundary Components Using Conformal Geometric Algebra

Dillon Pullano and Michael Barry (Canada)

Key words: Cadastre; Digital cadastre; Geoinformation/GI; Land management; Spatial planning; Topology; 3-D cadastre; Conformal geometric algebra

SUMMARY

As urban centers continue to grow and develop, there is an increasing need for institutions to be able to digitally model and perform legal boundary analysis on 3-D geospatial data. This paper reports on research that applies conformal geometric algebra (CGA) theory, objects and operations along with simple distance checks to build a computational model for performing topological analysis between 3-D legal boundary components. The scope is limited to testing the model on 3-D boundary points, lines and planes, which may define the boundary of a volumetric legal space such as a unit in a strata title scheme. The hypothesis is tested that the theoretical model can identify and classify topological relationships existing between 3-D boundary components being modelled in the context of a digital 3-D cadastre. A literature search suggests that the theory and methodology proposed have not been applied to the topological classification of 3-D boundaries as it has been applied here before.

A three-stage testing process is as follows. Firstly, the relationships that can exist between 3-D boundary component pairs are derived and categorized into different scenarios using CGA object concepts and theory. Categories include two 3-D boundary component CGA objects being parallel, collinear, coplanar or intersecting. Secondly, analytic testing hypotheses are posited for determining which CGA scenario is occurring between each component pair, and how to finalize the topological relationship for each scenario. Thirdly, these hypotheses are then tested and validated against simulated spatial datasets that represent the extent of relationships that can be classified between each component pair. The 3-D legal boundary components modelled in this paper are points with [x, y, z] coordinates, straight lines bound by two end points, and flat planes bound by at least three lines. The topological relationships that this research aims to identify are if two 3-D boundary components ‘Touch’, ‘Intersect’, or are ‘Disjoint’ from each other.

The 3-D simulation data and topological classification testing results presented in this paper are limited to those between point-point, line-point, line-line, plane-point, plane-line, and plane-plane boundary component pairs. Results support the mathematical testing procedures proposed to classify the topological relationship between different 3-D boundary component pairs. By first checking point, line, and face CGA objects against each other for being parallel, collinear, coplanar, or intersecting using CGA topological operators, the complexity of the remaining topology classification process can be reduced to point-point and line-point distance checks in many situations.

This paper covers the first phase of a research project. The model will be expanded to include classifying the topological relationships between the 3-D boundary components and a 3-D solid object defined by four or more faces. It will then be applied to 3-D property units defined by a strata survey plan to explore its application regarding 3-D cadastral object management. Being able to digitally model and perform analyses on these 3-D legal objects would increase the access to spatial information regarding current and future land and property developments and appraisals.