Vertical Displacement Analysis Based on Application of Univariate Model for Several Chosen Estimation Methods

Robert DUCHNOWSKI, Poland

Key words: Deformation measurement, engineering survey, estimation theory; robust estimation

SUMMARY

Considering a levelling network, which is established for vertical displacement analysis, we usually analyse several object points and a few reference points. The natural approach in such a case is to apply the multivariate functional models where parameter vector consists of the heights of the network points. However, some estimation methods, for example some variants of R-estimation, require another approach which is generally based on the univariate model. In such a case every object point is analysed separately from the others. The approach based on the univariate functional model can also be applied to the other estimation methods. Here, the following methods are considered: the least squares method (LSE), the Huber method (HE, the example of robust M-estimation), the Hodges-Lehmann weighted estimation (HLWE, the example of R-estimation) and two variants of M_{split} estimation, namely the squared M_{split} estimation (SMSE) and the absolute M_{split} estimation (AMSE). The analysis is based on an example levelling network and Crude Monte Carlo simulations (MC). For LSE and HE both types of the functional model are applied. For the rest of the methods only the univariate model is used. Since some methods are regarded as robust against outliers one can consider variant without outliers as well as several variants with outlying observations. The paper focuses on the accuracy of the estimations in question and how such an accuracy can be affected by different outliers. To investigate better how outlying observations might influence the estimates, empirical influence functions (EIF) are also determined. Generally, the analysis based on the univariate model is more sensitive to the location of outlier; the estimation results depend also on the point location, namely the network structure. As for the estimation accuracy, the multivariate model seems a better choice; however, the results of the univariate approach are at least comparable in some variants analysed. The conclusions resulting from the analysis of EIFs obtained are more varied. In many variants the univariate approach yields better results, namely results that are less sensitive to the growing outlier. It is also interesting the AMS estimation usually predominates over SMS estimation, and in many cases, it seems to be the best solution overall. Summing up, the univariate approach to vertical displacement analysis can be an alternative or a supplementation to the more traditional approach based on multivariate functional models.

Vertical Displacement Analysis Based on Application of Univariate Model for Several Chosen Estimation Methods

Robert DUCHNOWSKI, Poland

1. INTRODUCTION AND MOTIVATION

Deformation analysis is one of the most important tasks of surveying engineering. Generally, it is a very complex problem, hence several different approaches, measurement techniques and computation methods were developed (e.g., Caspary 2000; Wiśniewski 2009; Hekimoglu et al. 2010; Duchnowski 2010; Amiri-Simkooei et al. 2017; Nowel 2019). Usually the choice of the computation method type depends on measurement technique, network structure and type of the displacement analyzed. Considering a levelling network, which is established for vertical displacement analysis, we usually analyze several object points and a few reference points. The natural approach in such a case is to apply the multivariate functional models where parameter vector consists of the heights of the network points. In such a case the network is analyzed a whole. Another option is to analyze each point separately from the others, which leads to application of univariate model with only one parameter, namely the height of the point which displacement is analyzed. In such a case one tries to detect and assess vertical displacement of a single network point of the basis on several chosen observations. Such an approach leads to limitation of the number of observations which are applied for a single point analysis. The approach presented here is a natural one for the Hodges-Lehmann estimators (the basic R-estimators). In such a case the displacement is estimated on the base of at least two sets of the computed height of the point. These heights are computed in independent ways by applying the raw observations from at least two epochs respectively (Duchnowski 2010; 2013). Note that in the approach in question, one can increase the number of observations applied and hence increase the reliability and accuracy of the estimates (Wyszkowska and Duchnowski 2018).
The application of the univariate model has some advantages over the multivariate models, for example, it is advisable in detecting outliers which might occur and disturb the estimation process (e.g., Hekimoglu et al. 2014). Note that if an observation set includes some outliers then deformation analysis becomes much more complex, and sometimes it is just impossible to separate gross errors from displacements (see, e.g., Shaorong 1990; Duchnowski 2011). The main goal of the paper is to compare multivariate model versus univariate model applied in deformation analysis. The several following estimators, which are applied in deformation analysis, are considered: the least squares estimation (LS), an example of robust M-estimation – the Huber method (H), an example of R-estimation – the Hodges-Lehman weighted estimation (HLW), and two variants of M_{split} estimation: the squared M_{split} estimation (SMS) and the absolute M_{split} estimation (AMS). For the first two methods both multivariate and univariate models are applied, for the rest methods only the univariate model is used. The comparison is performed for the simulated leveling network.

2. MODELS AND METHODS APPLIED

Consider the following classical functional model of geodetic observations

$$\mathbf{y} = \mathbf{A}\mathbf{X} + \mathbf{v} \tag{1}$$

where: $\mathbf{y} = [y_1, \cdots, y_n]^T$ is an observation vector; $\mathbf{X} = [X_1, \cdots, X_r]^T$ is parameter vector; $\mathbf{v} = [v_1, \cdots, v_n]^T$ is vector of random errors, and $\mathbf{A} \in R^{n,r}$ is a known coefficient matrix. Such models are the basis for deformation analysis, namely for determining the shifts $\Delta\mathbf{X}_{(k,l)} = \mathbf{X}_l - \mathbf{X}_k$ between the epochs *l* and *k* (for example, the changes of the point coordinates between such epochs). Without loss of generality, we can consider two epochs thus $k = 1$ and $l = 2$. Let us also assume the following form of the covariance matrix of the observations

$$\mathbf{C}_{\mathbf{Y}} = \sigma_0^2 \mathbf{P}^{-1} \tag{2}$$

where: $\mathbf{P} \in R^{n,r}$ is a weight matrix of observations; σ_0^2 is a variance coefficient.

Deformation analysis can be based on several different estimation methods. First, we consider the classical LS estimator in the following form:

$$\hat{\mathbf{X}}_{LS} = \left(\mathbf{A}^T \mathbf{P} \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{P} \mathbf{y} \tag{3}$$

Such an estimator is the basis for the Huber estimate for which

$$\hat{\mathbf{X}}_{H} = \left(\mathbf{A}^T \overline{\mathbf{P}} \mathbf{A}\right)^{-1} \mathbf{A}^T \overline{\mathbf{P}} \mathbf{y} \tag{4}$$

where: $\overline{\mathbf{P}} \in R^{n,r}$ is an equivalent weight matrix. Such a matrix is computed in an iterative process. For each iterative step we can compute $\overline{\mathbf{P}}_{i,i} = \mathbf{P}_{i,i} \cdot w(\hat{v}_i)$, where $w(\hat{v}_i)$ is the weight function in the following form

$$w(\hat{v}_i) = \begin{cases} 1 & for \quad |\hat{v}_i| \le a \\ \dfrac{a}{|\hat{v}_i|} & for \quad |\hat{v}_i| > a \end{cases} \tag{5}$$

where: *a* – positive constant (usually assumed between 1.5 and 3.5). Note that the solution presented here is an iterative process which ends when the parameter vector is not changing between the iteration steps anymore (or the change is smaller than the assumed tolerance).

The Hodges-Lehman weighted estimates applied the test statistic introduced in (Duchnowski 2013), and which leads to the following direct estimate of the shift

$$\Delta\hat{\mathbf{X}}_{HLW} = \hat{\Delta}^{HLW} = medw\left(y_i - x_j\right) \tag{6}$$

where: *medw* is a weighted median operator. Considering application of this estimate in vertical displacement analysis, y_i and x_j are heights of the same object point at two different measurement epochs which are computed in independent ways by applying the heights of the reference points and the raw observation respectively (Duchnowski 2013; for other options see, Duchnowski 2010).

The last two estimates belong to the class of M_{split} estimators. They accept the split functional models (Wiśniewski 2009)

$$\mathbf{y} = \mathbf{A}\mathbf{X} + \mathbf{v} \quad \Rightarrow \begin{cases} \mathbf{y} = \mathbf{A}\mathbf{X}_{(1)} + \mathbf{v}_{(1)} \\ \mathbf{y} = \mathbf{A}\mathbf{X}_{(2)} + \mathbf{v}_{(2)} \end{cases} \tag{7}$$

The M_{split} estimates can be computed in the iterative process which can be described in the following way (Wiśniewski 2009, Wiśniewski et al. 2019)

$$\hat{\mathbf{X}}_{(1)} = \mathbf{D}_{(1)}(\hat{\mathbf{v}}_{(1)}, \hat{\mathbf{v}}_{(2)})\mathbf{y} \quad \hat{\mathbf{X}}_{(2)} = \mathbf{D}_{(2)}(\hat{\mathbf{v}}_{(1)}, \hat{\mathbf{v}}_{(2)})\mathbf{y} \tag{8}$$

where

$$\begin{aligned} \mathbf{D}_{(1)}(\hat{\mathbf{v}}_{(2)}) &= [\mathbf{A}^T\mathbf{w}_{(1)}(\hat{\mathbf{v}}_{(1)}, \hat{\mathbf{v}}_{(2)})\mathbf{A}]^{-1}\mathbf{A}^T\mathbf{w}_{(1)}(\hat{\mathbf{v}}_{(1)}, \hat{\mathbf{v}}_{(2)}) \quad \text{and} \\ \mathbf{D}_{(2)}(\hat{\mathbf{v}}_{(1)}) &= [\mathbf{A}^T\mathbf{w}_{(2)}(\hat{\mathbf{v}}_{(1)}, \hat{\mathbf{v}}_{(2)})\mathbf{A}]^{-1}\mathbf{A}^T\mathbf{w}_{(2)}(\hat{\mathbf{v}}_{(1)}, \hat{\mathbf{v}}_{(2)}) \end{aligned} \tag{9}$$

The weight matrices are as follows

$$\begin{aligned} \mathbf{w}_{(1)}(\mathbf{v}_{(1)}, \mathbf{v}_{(2)}) &= \text{Diag}\left(\ldots, w_{(1)}(v_{i(1)}, v_{i(2)}), \ldots\right) \\ \mathbf{w}_{(2)}(\mathbf{v}_{(1)}, \mathbf{v}_{(2)}) &= \text{Diag}\left(\ldots, w_{(2)}(v_{i(1)}, v_{i(2)}), \ldots\right) \end{aligned} \tag{10}$$

and they are computed by applying respective weight functions

$$w_{(1)}(v_{i(1)}, v_{i(2)}) = \frac{\psi_{(1)}\left(v_{i(1)}, v_{i(2)}\right)}{2v_{i(1)}} \quad \text{and} \quad w_{(2)}(v_{i(1)}, v_{i(2)}) = \frac{\psi_{(2)}\left(v_{i(1)}, v_{i(2)}\right)}{2v_{i(2)}} \tag{11}$$

where $\psi_{(1)}$ and $\psi_{(2)}$ are respective influence functions (see, Wiśniewski 2009, Wiśniewski et al. 2019). Of course, the whole process presented is iterative and it ends when the respective gradients of the objective functions are equal to zero. Note that different variants of M_{split} estimators differ from each other in the objective function, hence also in the influence and weight functions. Thus, for the squared M_{split} estimation (Wiśniewski 2009)

$$w_{(1)}(v_{i(1)}, v_{i(2)}) = p_i^2 v_{i(2)}^2 \quad \text{and} \quad w_{(2)}(v_{i(1)}, v_{i(2)}) = p_i^2 v_{i(1)}^2 \tag{12}$$

and for the absolute M_{split} estimation (Wyszkowska and Duchnowski 2019)

$$\begin{aligned} w_{(1)}(v_{i(1)}, v_{i(2)}) &= \begin{cases} -\dfrac{\left|v_{i(2)}\right|}{2v_{i(1)}} & for \;\; v_{i(1)} < 0 \\ \dfrac{\left|v_{i(2)}\right|}{2v_{i(1)}} & for \;\; v_{i(1)} > 0 \end{cases} \\ w_{(2)}(v_{i(1)}, v_{i(2)}) &= \begin{cases} -\dfrac{\left|v_{i(1)}\right|}{2v_{i(2)}} & for \;\; v_{i(2)} < 0 \\ \dfrac{\left|v_{i(1)}\right|}{2v_{i(2)}} & for \;\; v_{i(2)} > 0 \end{cases} \end{aligned} \tag{13}$$

Detailed description of the iterative process of M_{split} estimation can be found in (e.g., Wiśniewski 2009; Wiśniewski et al. 2019; Wyszkowska and Duchnowski 2020).

3. NUMERICAL TESTS

Let us now test the methods which are mentioned in the previous section in the case of an example simulated levelling network. First, the accuracy of estimates in question will be investigated. The tests will be done in an empirical way based on the Monte Carlo simulations.

Next the sensitivity to gross errors will be tested. In this context, empirical influence functions will be used.

3.1 Simulated network

Let us consider the leveling network which are presented in Fig. 1.

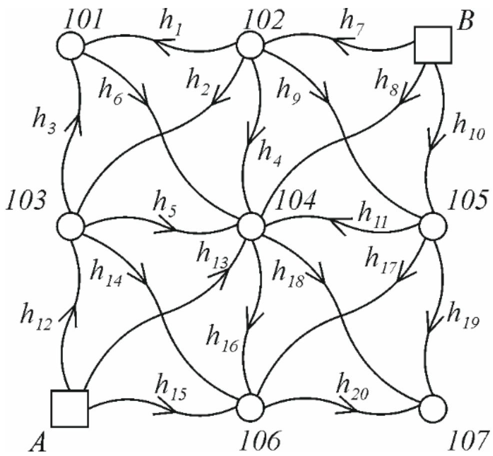

Fig. 1 Simulated levelling network

It is assumed that the reference points, namely *A* and *B*, are stable and $H_A = 0.000\,m,\ H_B = 0.000\,m$ at both measurement epochs. The assumed accuracy of the observations, namely the height differences between the network point h_i, is equal to 1 mm. Considering the displacement analysis with the multivariate model, the height differences are parameters in the model of Eq. (1) and they are estimated by adjustment of the epochs separately as for LS and H methods, see Eqs. (3 and 4). In the Huber method, the steering parameter $a = 2$. In the case of the univariate model we investigate only displacements of the points 104, 105 and 107. Because of the network structure, the analysis for the rest of the object points would give similar results. Thus, one considers two sets of heights differences computed for each object point under investigation and for each epoch in the following way

for point 104	for point 105	for point 107
$H_{104} = H_A + h_{13}$	$H_{105} = H_B + h_{10}$	$H_{107} = H_A + h_{15} + h_{20}$
$H_{104} = H_A + h_{12} + h_5$	$H_{105} = H_B + h_7 + h_9$	$H_{107} = H_B + h_{10} + h_{19}$
$H_{104} = H_A + h_{15} - h_{16}$	$H_{105} = H_A + h_{15} - h_{17}$	$H_{107} = H_B + h_8 + h_{18}$
$H_{104} = H_B + h_8$	$H_{105} = H_A + h_{13} - h_{11}$	
$H_{104} = H_B + h_{10} + h_{11}$		
$H_{104} = H_B + h_7 + h_4$		

Such computed heights are regarded as observations and used to estimate the point heights at measurement epochs and hence height differences (vertical displacements between the epochs). Of course, all such observations should have computed weights which create the new weight matrix. Such weights are computed by applying the variances obtained by application of the variances of the raw observations and the law of variance propagation. Thus, within the model of Eq. (1) we have $n_{104} = 6$, $n_{105} = 5, n_{107} = 3$ and $r = 1$. Considering M$_{split}$ estimation, one should assume the starting point of the iterative process. Here, we assume $\hat{\mathbf{X}}_{LS} + 0.01\,m$ for SMS estimation and two starting points, namely $\hat{\mathbf{X}}_{LS} - 0.01\,m$ and $\hat{\mathbf{X}}_{LS} + 0.01\,m$ as for AMS estimation (see, Wyszkowska and Duchnowski 2019, 2020).

3.2 Accuracy analysis

Assessment of the accuracy of the estimates under investigation is performed by applying Monte Carlo simulations and the root-mean-square deviation (RMSD)

$$\mathrm{RMSD}(\Delta\hat{H}_k) = \sqrt{\sum_{i=1}^{n} \frac{\left(\Delta\hat{H}_{k,i}^{MC} - \Delta H_k\right)^2}{n}} \tag{14}$$

where ΔH_k is theoretical assumed displacement of the *k*th point; $\Delta\hat{H}_{k,i}^{MC}$ is an estimate of the displacement of the *k*th point obtained at *i*th simulation. The number of simulations $n = 1000$, which seems enough in the context of the paper goal. To investigate the problem in a more general way we assume two variants of the point displacements:

$$\text{Variant I}: \Delta H_{104} = -0.030\,m,\ \Delta H_{105} = -0.015\,m,\ \Delta H_{105} = 0.010\,m,$$

$$\text{Variant II}: \Delta H_{104} = 0.003\,m,\ \Delta H_{105} = 0.002\,m,\ \Delta H_{105} = 0.005\,m,$$

The empirical accuracies of all estimators and for the both variants are presented in Table 1. Please note that the estimator with application of the multivariate model are marked with the additional M and those with the univariate model are marked with U.

Table 1. RMSDs of the displacement estimates (*mm*)

Variant	Parameter	Theoretical displacement	LS-M	LS-U	H-M	H-U	HLW	SMS	AMS
Variant I	ΔH_{104}	–30	0.66	0.69	0.71	0.70	0.72	0.73	0.79
	ΔH_{105}	–15	0.80	0.88	0.87	0.88	0.96	0.96	1.12
	ΔH_{107}	10	1.00	1.14	1.13	1.17	1.23	1.17	1.27
Variant II	ΔH_{104}	3	0.67	0.69	0.73	0.70	0.73	0.98	0.77
	ΔH_{105}	2	0.81	0.90	0.88	0.90	0.95	1.29	0.92
	ΔH_{107}	5	1.02	1.18	1.10	1.18	1.25	1.28	1.25

The results presented in Table 1 are similar to one another in many cases. Generally, LS as well as Huber estimates seem more accurate when the multivariate model is applied; however, in the case of Huber estimates there are some exceptions. Such a conclusion seems natural consequence of the number of observations involved. In the multivariate model, one uses all observations, hence the number of observations is bigger. Considering the methods which use only the univariate model, HLWE seems to have the best accuracy. In the case of M_{split} estimations, SMS estimates are more accurate than AMS estimates for the bigger assumed displacements (Variant I). When the displacements are small then the opposite is true.

Now let us investigate how some deterministic errors would influence the accuracy of the estimates. Thus, let some observations at the second measurement epoch be affected by gross errors. The outliers are chosen in three versions, the first one considers outlying observation which is relatively far from the object points analyzed; the second one, outlier is close to such points; the third one, both such outliers occur. The variants considered here as well the empirical accuracies obtained are presented in Table 2.

Table 2. RMSDs of the estimates when the observation set is affected by gross errors (*mm*)

Variant	Parameter	Theoretical displacement	LS-M	LS-U	H-M	H-U	HLW	SMS	AMS
Variant I with h_1^{II}+5 mm	ΔH_{104}	–30	0.69	0.69	0.72	0.70	0.73	0.74	0.81
	ΔH_{105}	–15	0.79	0.87	0.85	0.87	0.95	0.92	1.07
	ΔH_{107}	10	0.99	1.12	1.07	1.12	1.21	1.15	1.22
Variant I with h_{15}^{II}+5 mm	ΔH_{104}	–30	0.94	0.92	0.77	0.87	0.83	0.80	0.86
	ΔH_{105}	–15	1.13	1.36	0.93	1.36	1.21	1.05	1.18
	ΔH_{107}	10	1.51	2.10	1.21	2.10	1.80	2.81	1.75
Variant I with h_1^{II}+5 mm and h_{15}^{II}+5 mm	ΔH_{104}	–30	1.08	0.95	0.79	0.89	0.84	0.80	0.84
	ΔH_{105}	–15	1.08	1.32	0.91	1.32	1.17	1.00	1.12
	ΔH_{107}	10	1.54	2.07	1.19	2.07	1.79	2.78	1.77
Variant II with h_1^{II}+5 mm	ΔH_{104}	3	0.70	0.70	0.73	0.71	0.74	0.96	0.79
	ΔH_{105}	2	0.83	0.91	0.87	0.91	0.97	1.25	0.91
	ΔH_{107}	5	1.02	1.16	1.08	1.16	1.21	1.24	1.21
Variant II with h_{15}^{II}+5 mm	ΔH_{104}	3	0.95	0.94	0.76	0.88	0.84	2.11	0.84
	ΔH_{105}	2	1.14	1.34	0.94	1.34	1.19	3.34	1.38
	ΔH_{107}	5	1.44	2.02	1.15	2.02	1.75	3.36	2.61
Variant II with h_1^{II}+5 mm and h_{15}^{II}+5 mm	ΔH_{104}	3	1.09	0.95	0.80	0.90	0.86	2.14	0.84
	ΔH_{105}	2	1.07	1.32	0.89	1.32	1.19	3.30	1.39
	ΔH_{107}	5	1.45	1.98	1.12	1.98	1.72	3.38	2.67

The results show that the gross errors influence the accuracy of all the estimates tested. The impact of the outliers is the smallest when such an observation is far from the object point tested. Note that even LS estimates seem not to be affected by such an outlier. Considering LS and H estimates, the application of the multivariate model seems to be the better choice. This stems from the higher redundancy of such an observation set. However, for the point 104, the application of the univariate model seems advisable. For this point the estimated point displacement have better accuracy when such a model is applied, especially in the case of LS estimates. This of course stems from the fact that for this point the observation set created for the univariate model includes the biggest number of observations. For the rest of the estimates, one can note that the accuracy of HLWEs depends on the number of observations in the observation set. It increases with growing number of the observations. As for M_{split} estimates, accuracy of SMS estimates seems very dependent on the magnitude of the point displacements. For the bigger simulated displacement such estimates are usually slightly better that AMS estimates. For the smaller displacements, the opposite is true. Finally, it seems interesting than even the methods which are not regarded as robust seem not affected by a single outlier. This might result from a relatively small value of the gross error. To investigate such expression, one can perform the computations for the bigger gross error. An example results are presented in Table 3.

Table 3. RMSDs of the estimates when the observation set is affected by gross errors (*mm*)

Variant	Parameter	Theoretical displacement	LS-M	LS-U	H-M	H-U	HLW	SMS	AMS
Variant I with h_1^{II}+30 mm	ΔH_{104}	–30	1.22	0.70	0.72	0.70	0.74	0.73	0.79
	ΔH_{105}	–15	0.89	0.90	0.86	0.90	0.97	0.97	1.10
	ΔH_{107}	10	1.05	1.12	1.06	1.12	1.19	1.16	1.21
Variant I with h_{15}^{II}+30 mm	ΔH_{104}	–30	4.05	3.83	0.74	0.77	0.85	0.76	0.81
	ΔH_{105}	–15	4.70	6.03	0.89	6.03	1.22	3.92	1.13
	ΔH_{107}	10	6.25	9.99	1.14	9.99	1.84	24.62	28.46

The results show that the bigger gross error influences the estimation results in a much more significant way. Note that only the Huber method with application of the multivariate model and HLWEs have accuracy which is like the accuracy obtained for smaller gross error, respectively. It is also worth noting that both M_{split} estimates have failed for the estimation of the displacement of the point 107; however, the displacements of the points 104 and 105 are estimated with the acceptable accuracy. This shows how gross error might influence such estimates in an unexpected way.

3.3 Sensitivity of the estimate to gross errors

This section focuses on testing sensitivity of the estimates to gross errors. The best way is such a context is to apply empirical influence functions, EIFs (e.g., Duchnowski 2011;

Duchnowski and Wyszkowska 2020). For the goal of this paper we can apply the following form of EIF

$$\mathrm{EIF}(x) = T_n\left(\mathbf{y}_1, \mathbf{y}_2 + \mathbf{g}_2\right) \tag{15}$$

where: T_n – a tested estimator, $\mathbf{g}_l$ – vectors including gross errors at the respective epoch in the form

$$\mathbf{g}_2 = \left[g_{2,1} \quad \cdots \quad x_k \quad \cdots \quad g_{2,n}\right]^T \tag{16}$$

Thus, one can consider several constant gross errors, $g_{2,i}$, and one gross error, x_k, which values increases within the assumed interval. In such a way we can compute many different variants of EIFs which differ from each other in locations of gross errors in question.

Let us consider the variants which are discussed in the previous section. The respective EIFs are presented in Figures 2-9.

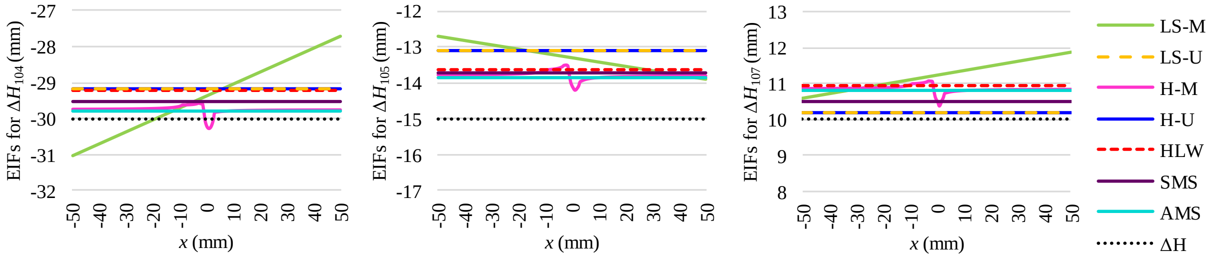

Fig. 2. EIFs, Variant I with x_1

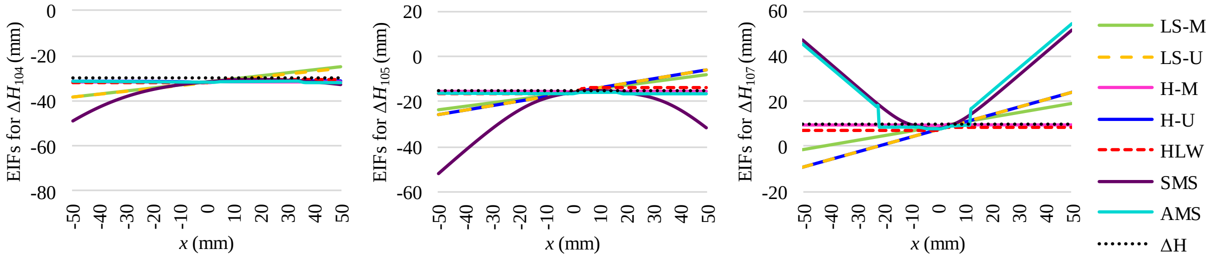

Fig. 3. EIFs, Variant I with x_{15}

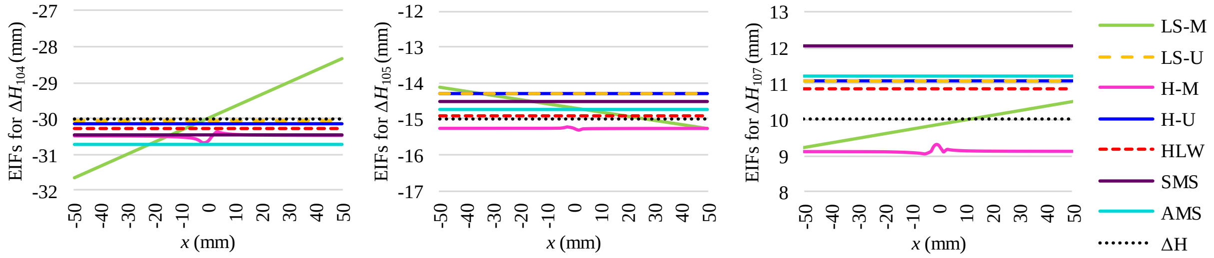

Fig. 4. EIFs, Variant I with x_1 and $g_{2,15} = 5\,mm$

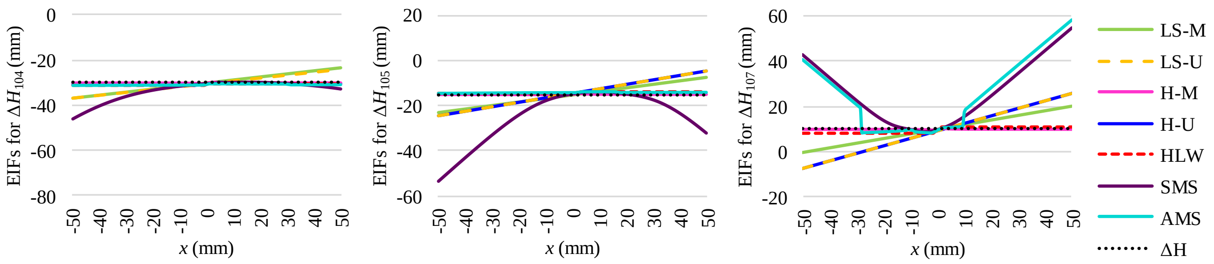

Fig. 5. EIFs, Variant I with x_{15} and $g_{2,1} = 5mm$

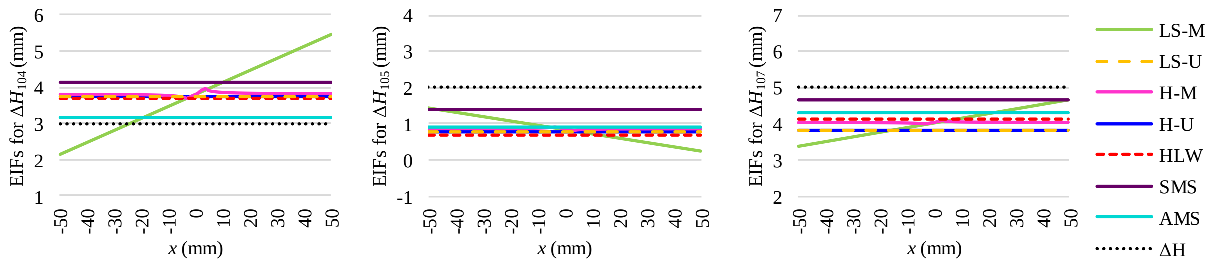

Fig. 6. EIFs, Variant II with x_1

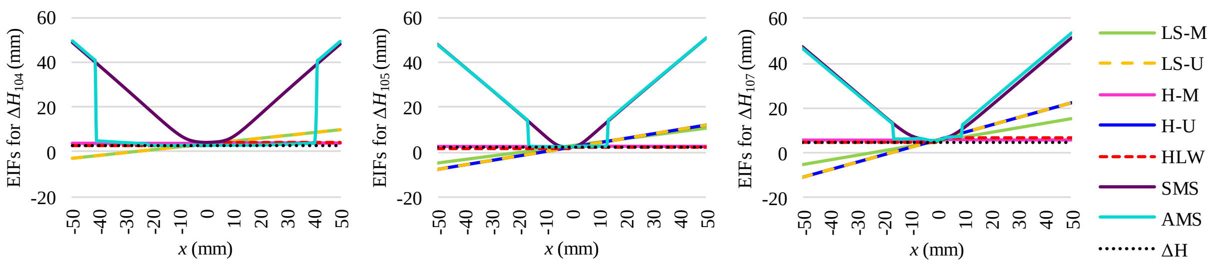

Fig. 7. EIFs, Variant II with x_{15}

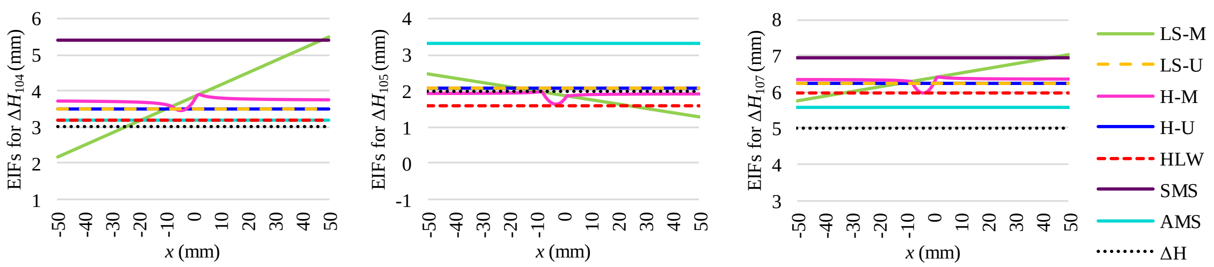

Fig. 8. EIFs, Variant II with x_1 and $g_{2,15} = 5mm$

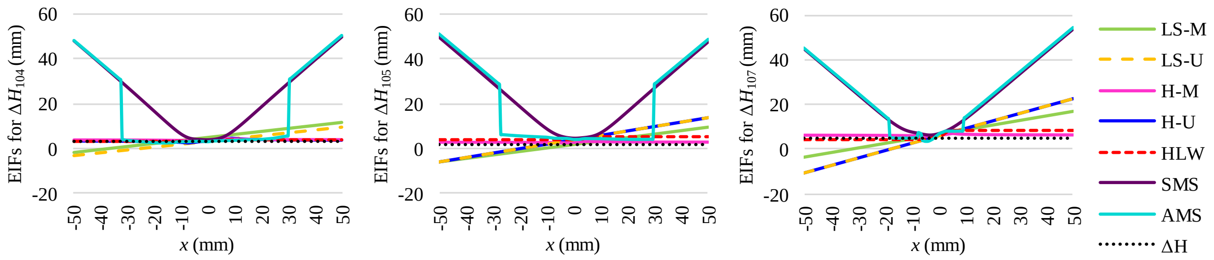

Fig. 9. EIFs, Variant II with x_{15} and $g_{2,1} = 5\,mm$

Analysis the EIFs obtained allows us to draw some interesting conclusions concerning the sensitivity of the estimate. LS estimates obtained with application of the univariate model are usually less sensitive to the gross errors, especially to the single gross error. The estimates of the Huber method are less sensitive when the multivariate model is applied. Note that in the case of the univariate model EIFs of the Huber method are the same as the respective EIFs of LS estimation as for the points 105 and 107. Thus, one can conclude that the method does not identify the outliers. HLWEs seem insensitive to gross errors at all. The EIFs of these estimates are very often horizontal lines, thus increasing gross error does not influence the estimation process in a more extensive way. The EIFs obtained for SMS and AMS estimates also provide valuable information. The results obtained for the cases in which the increasing gross error affects the observation h_{15}^{II} seem especially interesting (Figs. 3, 5, 7, 9). They show that AMS estimates are much less sensitive to the gross errors. In many cases considered, such estimates are almost insensitive to small or moderate values of the gross error. However, for the biggest value of the gross error, the response of SMS and AMS estimates are often similar to each other.

4. CONCLUSIONS

The paper addresses the application of two types of possible functional models in deformation analysis. Analysis of network point displacements is usually based on application of the multivariate model. However, for some estimates, for example HLWE, the univariate model is a natural choice. The empirical tests reveal pros and cons for both models. As for LS method and especially the Huber method, the multivariate model seems a better choice. Application of the univariate model usually decreases the redundancy of the observation set and hence makes the estimates more sensitive to outliers. This is especially important in robust M-estimation where sometimes the iterative process cannot start at all, and the outliers stay undetected. However, application of the univariate model might be sometimes advisable. This concerns cases where outlier in not involved in computing coordinates which are included in the observation sets for the application of the univariate model. Then, the estimate of the point displacement would be free of the bad influence of outlier. In such a context estimates obtained with application of the univariate model can be regarded as a supplementation (or confirmation) of the results obtained with the application of the multivariate model.

Application of the univariate model does not mean the results will be more sensitive to gross errors or less accurate. HLWE, which applies the univariate model, has the accuracy which is comparable with the accuracy of the estimates based on the multivariate model. Its sensitivity to outlier is also high. What is more, such a sensitivity does not depend on the magnitude of the gross errors (in the case of the other estimates such a relation is obvious). Finally, the test show that AMS estimation is less sensitive to gross error than SMS estimation, which confirms previous conclusions for the multivariate model (Wyszkowska and Duchnowski 2019). However, in some cases, SMS are more accurate than AMS estimates, thus, the choice of the better M_{split} estimate is not so obvious and easy.

REFERENCES

Amiri-Simkooei, A.R., Alaei-Tabatabaei, S.M., Zangeneh-Nejad, F. and Voosoghi, B., 2017, Stability analysis of deformation-monitoring network points using simultaneous observation adjustment of two epochs. Journal of Surveying Engineering, Vol. 143, No. 1, Article No. 04016020.

Caspary, W.F., 2000, Concepts of network and deformation analysis, University of New South Wales, Kensington.

Duchnowski, R., 2010, Median-based estimates and their application in controlling reference mark stability, Journal of Surveying Engineering, Vol. 136, pp. 47–52.

Duchnowski, R., 2011, Robustness of the strategy for testing levelling mark stability based on rank tests, Survey Review, Vol. 43, pp. 687–699.

Duchnowski, R., 2013, Hodges-Lehmann estimates in deformation analyses, Journal of Geodesy, Vol. 87, No. 10, pp. 873–884.

Hekimoglu, S., Erdogan, B. and Butterworth, S., 2010, Increasing the efficacy of the conventional deformation analysis methods: alternative strategy, Journal of Surveying Engineering, Vol. 136, pp. 53–62.

Hekimoglu, S., Erdogan, B., Soycan, M. and Durdag, U.M., 2014, Univariate approach for detecting outliers in geodetic networks, Journal of Surveying Engineering, Vol. 140, No. 2, Article No. 04014006.

Nowel, K., 2019, Squared $M_{split(q)}$ S-transformation of control network deformations, Journal of Geodesy, Vol. 93, pp. 1025–1044.

Shaorong, Z., 1990, On separability for deformations and gross errors, Journal of Geodesy, Vol. 64, No. 4, pp. 383–396.

Wiśniewski, Z., 2009, Estimation of parameters in a split functional model of geodetic observations (M_{split} estimation), Journal of Geodesy, Vol. 83, No. 2, pp. 105-120.

Wiśniewski, Z., Duchnowski, R. and Dumalski, A., 2019, Efficacy of M_{split} estimation in displacement analysis, Sensors, Vol. 19, No. 22, Article No. 5047.

Wyszkowska, P. and Duchnowski, R., 2018, Increasing reliability of basic R-estimates in deformation analysis, Proceeding of 2018 Baltic Geodetic Congress, IEEE, pp. 241-245.

Wyszkowska, P. and Duchnowski, R., 2019, M_{split} estimation based on L_1 norm condition, Journal of Surveying Engineering, Vol. 145, No. 3, Article No. 04019006.

Wyszkowska, P. and Duchnowski, R., 2020, Iterative process of $M_{split(q)}$ estimation, Journal of Surveying Engineering, in press.

BIOGRAPHICAL NOTES

Robert Duchnowski, PhD habil., Head of Department of Geodesy, Institute of Geodesy and Civil Engineering, Faculty of Geoengineering, University of Warmia and Mazury in Olsztyn, Poland. Field of interest: theory of estimation, robust estimation in geodetic computations, deformation analysis.

CONTACTS

prof. Robert Duchnowski
Department of Geodesy, Institute of Geodesy and Civil Engineering, Faculty of Geoengineering, University of Warmia and Mazury in Olsztyn
Oczapowskiego 1 Street
Olsztyn
Poland
Email: robert.duchnowski@uwm.edu.pl