Varying Geometry Area Determination of Selected Sites in Rivers State University, Port Harcourt, Nigeria

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Key words: Area, Geometry, Perimeter, Shape and Variations

SUMMARY

In land parcellation exercise computing area of parcel is an important process of obtaining surveying deliverables. The aforesaid is a critical activity associated with land management which underscores that shape and size of land determines its use. The neglect of geometry is the contributing factor to variations in size and the resultant effect to prospective vendees paying more for less value because land is mostly parcellated based on the perimeter without due consideration to its geometry. This research investigates the variations in the area of each parcel of land with varying geometry. A sample of 18 varying parcels; nine 30 by 45m and nine 100 by 100m were traversed using classical method with same traverse length. The computed boundary coordinates were adjusted using least squares, and the area of each parcel was computed and the plans of the different parcels were superimposed on each other. The angular misclosures ranges from -1" to 39.6" and -6" to 58.38" with linear accuracies of 1:9,000 to 1:228,000 and 1:8,000 to 1:232,000. The difference in area from the variations in the geometry ranges from 5.339m^2 to 326.238m^2 and 38.129m^2 to 2344.237m^2 on the smaller and larger parcels respectively. The adjusted coordinates of each parcel with their standard errors were computed using least squares; after which the areas of each parcel were computed and the difference in area between the unadjusted and adjusted coordinates were also computed, hence geometry has obvious impact on the area of any landed property.

Palabras clave: Área, Geometría, Perímetro, Forma y Variaciones

SUMMARY (Spanish)

En el ejercicio de parcelación de tierras, el área de computación de parcela es un proceso importante para obtener entregables de topografía. Lo anterior es una actividad crítica asociada con la gestión de la tierra que subraya que la forma y el tamaño de la tierra determinan su uso. El descuido de la geometría es el factor que contribuye a las variaciones en el tamaño y el efecto resultante para los posibles vendeos que pagan más por menos valor porque la tierra se parcela principalmente en función del perímetro sin la debida consideración a su geometría. Una muestra de 18 parcelas diferentes; Nueve de 30 por 45 m y nueve de 100 por 100 m se atravesaron utilizando el método clásico con la misma longitud de recorrido. Las coordenadas de límite calculadas se ajustaron utilizando mínimos cuadrados, y el área de cada parcela se calculó y los planos de las diferentes parcelas se superpusieron entre sí. Los errores de cierre angulares varían de -1" a 39.6" y de -6" a 58.38" con precisiones lineales de 1:9.000 a 1:228.000 y de 1:8.000 a 1:232.000. La diferencia de área de las variaciones en la geometría varía de 5.339m2 a 326.238m2 y de 38.129m2 a 2344.237m2 en las parcelas más pequeñas y más grandes, respectivamente. Las coordenadas ajustadas de cada parcela con sus errores estándar se calcularon utilizando mínimos cuadrados; Después de lo cual se calcularon las áreas de cada parcela y también se calculó la diferencia de área entre las

coordenadas no ajustadas y ajustadas, por lo tanto, la geometría tiene un impacto obvio en el área de cualquier propiedad de tierra.

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1.1 Background to the Study

The extent of a land can be measured in square meters, acre or hectares. The area of any landed property is the quantity that expresses the extent of a two-dimensional (2D) figure or shape. The imperial and metric systems are the two standards of measurements used. When using the imperial system measurements are obtained in inches, feet and square yards or acres, while the metric system measures in centimeters (cm), meters (m), square meters (m^2) or hectares (ha) when area is more than 10,000m^2 (Awosusi, 2020). In plane surveying, area refers to the orthographic projection of a tract of land on a horizontal plane (as such the curvature of the earth is negligible). Area determination is one of the primary objectives of land surveys activities, hence survey deliverables (size, shape and position) is required for the title documents of land in general. Also, in civil engineering works such as designing of bridges, dams, and reservoirs, the area of the catchment of a river special survey are required. Furthermore, the areas of fields are also required for planning and management of a study. Area computation is indeed an important part of the office work involved in surveying (Chandra, 2006). A plot is an arbitrary term used to describe a land division made in a particular area (Awosusi, 2020). In south eastern and Southern Nigeria, land division of 50×100ft (15.24m x 30.48m) is the widely used size for a plot of land, but this dimension differs according to the location of that landed property within Nigeria but due to the variations in geometry, a plot can range from 450sqm to 1200sqm. Therefore, it is important to always enquire from land vendors on what the size of a plot for sale is before proceeding with any financial transaction.

Initially, plots were approximately 1000Sqm but as the prices of land increased, some plots are now sold at 400sqm or less in Government Reserved Areas (GRA's) in Enugu, Kano, Kaduna, Lagos, Port Harcourt and other major cities. In Calabar, new layouts offer up to 1,100Sqm for commercial plots (Awosusi, 2020). Since, Nigeria uses the metric system of measurement; plots of land are usually described in square meters. Land parcellation is a critical activity associated with land management. This process underscores the shape and extent which determines land use, to satisfy the requirements, the angular parameter with respect to the extent (area) becomes very imperative. During subdivision or purchase of any landed property, the shape of each parcel should be taken into consideration so as to maximize the use of that land. Geometry is a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solids. A point is a location in space, often symbolized by a dot. A line consists of many points that follows a straight path and extends indefinitely in both directions. A line segment is a part of a line between two distinct end points. A ray is a part of a line that includes an end point and all points on one side of the end point. A ray has only one end point, which is always written first. An angle is a geometric figure formed by two rays that share a common end point. The common end point is called the vertex of the angle. In naming an angle, the vertex must be the middle letter; sometimes an arc is drawn to illustrate the location of an angle (Miller, O'Neill, and Hyde, 2007).

The most common unit to measure an angle is the degree. Two rays that form a quarter turn of a circle make a 90° angle; a 90° angle is called a right angle. Two rays that form a half turn of a circle mark a 180° angle. A 180° angle is called a straight angle because it appears as a straight line. A full circle has 360°. For example, the second hand of a clock sweeps out an angle of 360° in one (1) minute. We can approximate the measure of an angle by using a protractor**.** A protractor uses equally spaced marks around a semicircle to measure angles from 0° to 180°. An angle is said to be acute if its measure is between 0° and 90°. An angle is said to be obtuse if its measure is between 90° and 180°. Two angles are said to be equal or congruent if they have the same measure. Two angles are said to be complementary if the sum of their measures is 90°, while two angles are said to be supplementary if the sum of their measures is 180° (Miller et al, 2007). Parallel lines lie on the same plane but never intersect. Two intersecting lines form four angles called vertical angles. They appear on opposite sides of the vertex. Vertical angles are equal in measure. If two lines intersect at a right angle, they are called perpendicular lines. With perpendicular lines, the vertical angles each measure 90°. If a third line intersects two parallel lines, that line is called a transversal (Miller et al, 2007).

A triangle is a three-sided figure whose sum of interior angles is 180°. A triangle may be categorized by the measures of their angles and by the number of equal sides or angles. In an acute triangle, all three angles are acute (less than 90°). In a right triangle, one of its angles is 90°. In an obtuse triangle, one angle is obtuse (greater than 90° but less than 180°). In an equilateral triangle, all three sides and all three angles are equal in measure. An isosceles triangle is a triangle in which two sides are equal in length, and the angles opposite the equal sides are also equal in measure. A scalene triangle is a triangle in which no sides or angle is equal in measure. A right triangle is a triangle with a 90° angle. The two sides forming the right angle are called the legs. The side opposite the right angle is called the 'Hypotenuse' and is always the longest side. For any right triangle, the Pythagorean Theorem gives the relationship among the length of the sides (Miller et al, 2007). A polygon is a flat figure formed by the line segments connected at their ends. A four-sided polygon is called a quadrilateral. A parallelogram is a quadrilateral with opposite sides parallel, and the opposite sides are equal in length. A rectangle is a parallelogram with four right angles. A square is a rectangle with sides of equal length. A Rhombus is a parallelogram with sides of equal length; the angles are not necessarily equal, as with a square. A Trapezoid is a quadrilateral with one pair of parallel sides.

The perimeter of a polygon could be obtained by adding the lengths of all sides. However, with some geometric figures we can shorten the process by using a formula. The area of a region is the number of square units that can be enclosed within the region (Miller et al, 2007). The summation of the internal angles of a polygon with n sides is (2n - 4) right angles, while the sum of the exterior angles is (2n + 4) 90° (Subramanian, 2012). A circle is a figure consisting of all points located on the same distance from a fixed point. The fixed point is called the centre of the circle. A radius of a circle is a line segment drawn from the centre of a circle to a point on the circle. The radius can be measured from the centre to any point on a circle. The diameter of a circle is a line segment connecting two points on a circle and passing through the centre of the circle. The length of a diameter is twice the radius. The distance around a circle is called a circumference. Early mathematicians discovered that the ratio of the circumference of a circle to its diameter is constant, and this is true for any circle. The constant value is given by c/d and it is called the number π (pi). The number π is about 22/7. The relationship between the circumference and diameter of a circle gives the formula of a circumference of a circle. Volume is the amount of space occupied by a three-dimensional object as measured in cubic units. Volume is another word for capacity (Miller et al, 2007).

In surveying, an angle is gotten from the algebraic difference between two directions. The angle between a line and the prolongation of the preceding line is known as deflection angle. Deflection angle may be on the left or right of the prolongation. 'Angles to the right' are referred to angles measured clockwise from the preceding line to the next line. Interior angles are angles between the adjacent lines of a closed polygon while the exterior angles are the explements of interior angles (Chandra, 2006). The primary methods used for land measurement include Theodolite or Total Station traversing and Real Time Kinematics (RTK) GPS survey. In surveying, traversing means determining the length and directions of consecutive lines (Duggal, 2007). The integrated measurement of distances and directions provides the coordinates for determining the area of any given land. With the method of traversing, the area of any parcel of land can be determined. The method of computation of area depends upon the shape of the boundary of the tract of land and the accuracy required. If the parcel is enclosed by straight boundaries, it can be subdivided into simple geometrical shapes, such as triangles, rectangles, trapezoids, etc., and areas of these figures are determined from the dimensions obtained from the survey plan (Dashe, 1987). When the boundaries are irregular, they are replaced by short straight boundaries, and the area is computed using approximate methods; but when the boundaries are very irregular, the area can be determined by means of a planimeter.

The degree of accuracy of the computed area depends on the accuracy of the field measurements, the accuracy of plotting when the calculations are made from a plan, and the method adopted for the computation (Chandra, 2006). The area of a parcel of land can be computed by cross coordinate method, double meridian distance or double latitude method. Area of an irregular boundary can be determined by taking perpendicular offset from a baseline to the irregular boundary and computed using either trapezoidal or Simpson’s rule. Area can also be determined by map measurements. This can be achieved by either of the following: running a planimeter over the enclosing lines; dividing the map into geometric shapes like squares, rectangle, triangle; digitizing coordinates and counting coordinate squares (Ghilani and Wolf, 2008).

In this study a closed traverse will carried out on different geometry of land. The variation is made such that the perimeter is held constant but the angles will be altered by 5^0 for each parcel. The baseline will also be held constant. Thereafter, the areas of the different parcels were computed while the extent or size of land is a factor for the determination of the value of land. People had gotten less of the value anticipated due to the issue of the size of the property, and they end up paying for more than they get. The non-adherence to the principles of geometry is the contributing factor to variations in extent, and the effect is that people have less than the value they have paid for. Generally, land is parcellated based on the perimeter without due consideration to its geometry. This has increased the possibilities of land in dispute, and had led to contention between adjacent land owners, and sometimes land vendors. Others who acquire a parcel of land from an already designed layout may experience some difficulty in positioning their facilities to face a particular direction, because of the shape of that landed property. Hence, the need for this study is to provide the dynamics of the varying geometry as it impacts on the area of any parcel of land.

The aim of this study is to determine the area of a parcel of land by varying geometry. The objectives are as follows;

1. To carry out traverse on different geometry of a land with the same perimeter and produce boundary coordinates.
2. To determine the adjusted coordinates using least squares and the area of each parcel.
3. To produce the plans of the different parcels superimposed on each other and to present both tabular and graphical form, the change in area as a result of the variations in angles.

The location of this study is at the right wing of the entrance to the Faculty of Environmental Sciences in Rivers State University. It is bounded by geographical coordinates: latitude 4^0 47’ 30”N to latitude 4^0 48’ 20”N and longitude 6^0 58’ 35”E to longitude 6^0 59’ 05”E. The study area is an open greenery and relatively has less human movement.

Figure 1: Study Area. Source: Google Earth imagery, 2021.

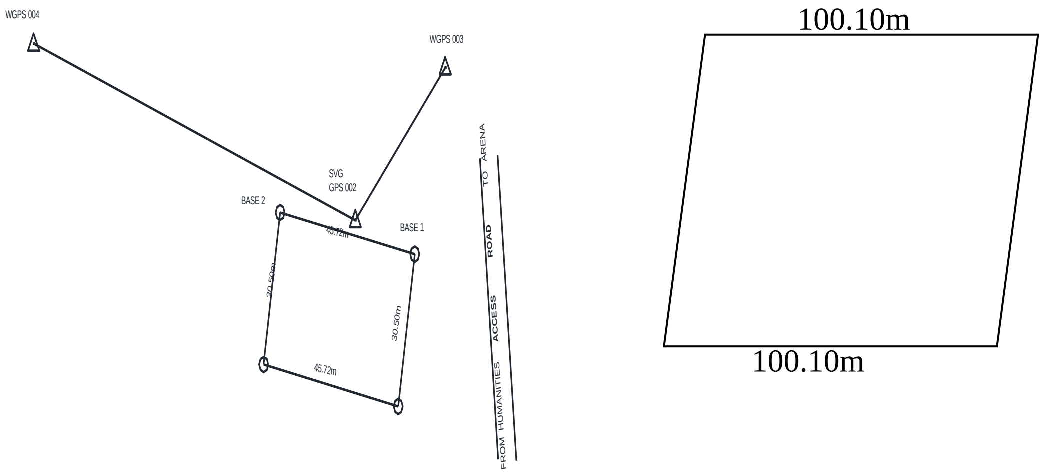

Figure 2: Schematic diagram of the Study Area.

2.0 Conceptual Framework

2.1 Types of Surveying

On the basis of whether the curvature of the earth is taken into account or not, Ejiobih (2008) identified that surveying can be divided into two main categories:

2.1.1 Plane surveying

This is the type of surveying where the mean surface of the earth is considered as a plane. All angles are considered to be plane angles (Dashe,1987). For areas less than 250km^2 plane surveying can safely be used for which the distances of this study are less than 1km (Ejiobih, 2008).

2.1.2 Geodetic surveying

This is that branch of surveying, which takes into account the true shape of the earth (spheroid)

2.2 Traversing

A traverse consists of a series of straight lines of known length related to one another by known angles. These lines connect successive established points and the points defining the ends of this traverse line are called traverse stations or traverse points (Dashe,1987; Ejiobih, 2008). The lengths of the lines are determined either by direct measurements using a tape or electronic distance measuring equipment (EDM) or indirect measurement using the method of tacheometry. The angles at the stations between the traverse lines are measured with a theodolite or a total station (Chandra, 2006). Traverse can be classified into two categories: open traverse and closed traverse.

2.2.1 Open Traverse

An open traverse originates at a point of known position but does not close at a known point. It is mathematically open and geometrically open (Dashe,1987; Chandra, 2006; Ejiobih, 2008). It consists of a series of lines that are connected but do not return to the starting point nor close in another known point. Open traverse is not ideal for any permanent studies because it doesn't reveal mistakes or errors. Open traverse may extend for long distances without the opportunity for checking the accuracy of the ongoing work. An open traverse is useful for providing controls and construction services for roads, pipelines, electricity transmission lines etc. however, in such cases, the observations should be carefully repeated to avoid mistakes.

2.2.2 Closed Traverse

Close traverse originates at a point of known position and close at another point of known position (Dashe,1987; Chandra, 2006; Ejiobih, 2008). This type of traverse provides computational checks allowing detection of systematic errors in both distance and directions. A closed traverse is mathematically a closed polygon and should satisfy the conditions that apply to a closed figure. The algebraic sum of the latitudes and departures in a closed traverse should be zero. The sum of the exterior angles of a closed traverse is equal to (2n + 4) right angles, (if external angles were observed); and (2n - 4)90° if internal angles were observed; where ‘n’ is the number of sides of the traverse (Dashe,1987; Chandra, 2006). If the deflection angles of the traverse are measured the sum of the deflection angles is equal to 360° irrespective of the number of sides. A closed loop traverse originates and terminates on the same points of known horizontal position. This type of traverse permits and internal check on the angular measurements but detection of systematic errors in linear measurements or arrows in the orientation of the traverse is not possible (Chandra, 2006).

2.3 Area Determination

Area can be obtained directly from field measurements or from maps as such area gotten from field measurement can be divided into two categories: area consisting of regular boundary and area consisting of irregular boundary (Dashe,1987; Chandra, 2006).

2.3.1 Area Consisting of Regular Boundary

This can be determined either by dividing the parcel into triangles, trapezoid or calculated from coordinates of the traverse stations (Dashe,1987).

2.3.1.1 Area of a Triangle: Allan (2004) identified that the area of a triangle whose lengths of sides are known especially in a three-dimensional coordinate system is Hero's formula expressed as:

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)} \qquad (1)$$

Where: a, b and c are the lengths of sides of the triangle; and $s = \frac{1}{2}(a + b + c)$

Another formula for calculating the area of a triangle is: $\text{Area} = \frac{1}{2}\, ab \sin C$ (2)

Where 'C' is the included angle between sides 'a' and 'b'

The choice of equations to use depends on the triangle parts that are most conveniently determined and which also depend on the nature of the area and the type of equipment available

The area of a right-angled triangle can be calculated as: $\text{Area} = \frac{1}{2}\, bh$ (3)

Where b and h are base and height respectively

2.3.1.2 Area of a trapezoid: The height of a trapezoid is the distance between the two parallel lines (Allan, 2004). The area of a trapezoid is given as:

$$A = \tfrac{1}{2}\,(\text{sum of the parallel sides})\ \text{height} = \tfrac{1}{2}\,(a + b)\, h \qquad (4)$$

Where: a, b are the two parallel sides and h is the perpendicular distance between the parallel sides. The area of a trapezoid is achieved by running a baseline and measuring the perpendicular offsets to salient points.

2.3.1.3 Area from Coordinates: The coordinates can be obtained by running a closed traverse (Dashe,1987; Allan, 2004; Chandra, 2006; Ejiobih, 2008). The area within the traverse can be computed from the coordinates obtained after balancing the closing error. The area can be computed by arranging the coordinates in the determinant form:

$$\begin{array}{ll} E_1 & N_1 \\ E_2 & N_2 \\ E_3 & N_3 \\ E_n & N_n \\ E_1 & N_1 \end{array}$$

Figure 3: Cross coordinate method of area determination

$$\text{Area} = \tfrac{1}{2}\,[(E_1N_2 - N_1E_2) + (E_2N_3 - N_2E_3) + \ldots + (E_nN_1 - N_nE_1)] \qquad (5)$$

Alternatively, the area can be computed as:

$$\text{Area} = \tfrac{1}{2}\,[(E_1N_2 + E_2N_3 + E_3N_n + E_nN_1) - (N_1E_2 + N_2E_3 + N_3E_n + N_nE_1)] \qquad (6)$$

2.3.1.4 Area by Double Meridian Distances (DMD) Method: This method of area determination requires balanced departures and latitudes of the tract's boundary lines, which are normally obtained in traverse computations (Dashe,1987; Chandra, 2006; Ejiobih, 2008). The Double Meridian Method (DMD) is useful for checking answers obtained by the coordinate method. By definition, the meridian distance of a traverse course is the perpendicular distance from the midpoint of the course to the reference meridian. This rule is applied when calculating DMDs: "The DMD for any traverse course is equal to the DMD of the preceding course, plus the departure of the preceding course, plus the departure of the course itself". Signs of the departures must be considered. The DMD of the first course is its departure. This implies that the meridian distance of any course of a traverse equals the meridian distance of the preceding course plus half the departure of the preceding course, plus half the departure of the course itself. DMDs equal to twice the meridian distances and a single division by 2 is made at the end of the computation. A check on all computations is obtained if the DMD of the last course (after computing the traverse), is equal to its departure with an opposite sign. The algebraic summation of all the double areas gives twice the area inside the entire traverse. Signs of the products of DMDs and latitudes must be considered.

DMD of AB = departure of AB

DMD of BC = DMD of AB + departure of AB + departure of BC (7)

2.3.2 Area consisting of irregular boundary

If the boundary of a parcel is irregular the area is computed using the offsets taken at a regular interval the area between the traverse line and irregular boundary may be determined by either of the following methods.

2.3.2.1 Mid-Ordinate Rule: In this method the tract is divided into segments and the offsets at the midpoint of each interval are taken. The measured offsets are the mid-ordinate.

Area = $d(m_1 + m_2 + m_3)$ (8)

Where: L = Length of the baseline,

n = number of segments and

$d = L/n$ (9)

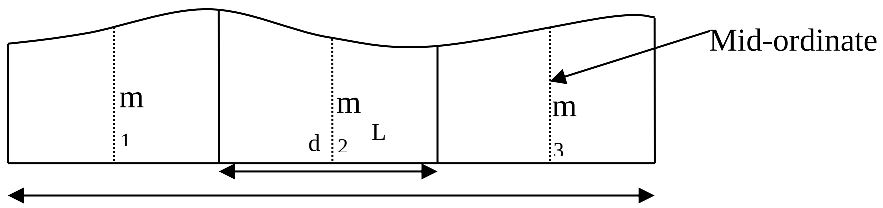

Figure 4: The mid-ordinate rule

2. Average Ordinate Rule: This method uses the average ordinates of the area (Dashe,1987). However, if the offsets are not taking at regular intervals, then the area of each trapezoid is computed separately and added together to obtain area.

Area = $d_1/2\,(O_1 + O_2) + d_2/2(O_2 + O_3) + d_3/2\,(O_3 + O_4)) + d_4/2\,(O_4 + O_5)$ (10)

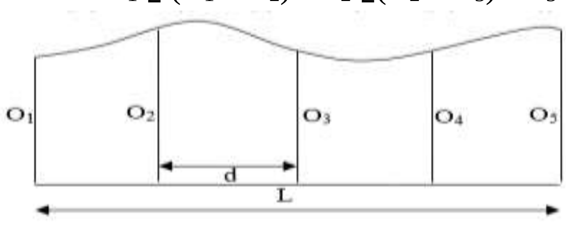

Figure 5: The mid-ordinate rule

3. Trapezoidal Rule: In this method the area is divided into trapezoids (Dashe,1987). The area of each trapezoid is determined and added to get the total area of the tracts.

$$\text{Area} = \frac{(O_1 + O_2)d}{2} + \frac{(O_2 + O_3)d}{2} + \frac{(O_3 + O_4)d}{2} + \frac{(O_4 + O_5)d}{2} \quad (11)$$

4. Simpson's Rule: In Simpson's rule, the boundary line segment between the ordinates is assumed to be a parabola or made up of parabolic arcs; this is also called the parabolic rule (Dashe,1987). This method should be preferred to the trapezoidal rule when the boundary line is curved and departs considerably from the straight line. Simpson's rule states that: 'To the sum of the first and last ordinates, add twice the sum of the third, fifth, seventh, etc. ordinates and four times the sum of the second, fourth, sixth, etc. ordinates and multiply this sum by x/3, where x is the distance between the ordinates. To apply Simpson's rule, we need to have even number of sections or odd number of ordinates (Subramanian, 2012).

$$\text{Area} = \frac{d}{3}\left[\sum(\text{end ordinates}) + 4\sum(\text{even ordinates}) + 2\sum(\text{odd ordinates})\right] \quad (12)$$

2.4 Least Squares Adjustment

Least squares method is used in the adjustment of measured quantities containing random errors (Wolf and Ghilani, 1997). It is assumed that all systematic errors have been removed from the observed values and that only random error and blunders which have escaped detection remain. Least squares adjustment technique makes use of redundant observation in the mathematical modeling of a given problem to minimize the sum of squares of discrepancies between the observation and the most probable value (Wolf and Ghilani, 1997). Least squares method is considered one of the best and common methods of adjustment computations when we have redundant observations or an over determined system of equations. The least squares solution can be obtained by two methods: method of variation of parameters or the observation method and method of correlates or the condition equation. In this study, the observation equation was adopted for the adjustment of the final coordinates.

2.4.1 Least Squares Adjustment by method of Observation Equation

The functional relationship of the observation and the adjusted parameters is given as:

$$L^a = F(X^a) = L^b + V \quad (13)$$

$$V = AX - L^b \quad (14)$$

Where: L^a = adjusted observation

V = Residual vector

A = Design matrix

L^b = Original observation

X = Adjusted parameter

Solution of the unknown parameters or the estimate of the correction of approximate parameter vector, X, in a non-linear model is given as:

$$L^a = f(x^a) = f(x^0 + x) \quad (15)$$

$$x^a = x^0 + x \quad (16)$$

Where: x^0 = Approximate value of adjusted parameters

X = Correction to approximate value

$$X = -(A^T PA)^{-1} A^T PL \tag{17}$$

Where: $(A^T PA)$ = normal equation coefficient matrix, $A^T PL$ = normal equation constant and P = unit weight

$$\sum_X = \sigma_0^2 (A^T PA)^{-1} \text{ = variance – covariance matrix of adjusted parameters} \tag{18}$$

Where: $\sigma_0^2 = V^T PV/n\text{-}m$ = aposteriori of unit weight (19)

n = number of equation and m = number of parameters

Having obtained the variance-covariance matrix of X, the standard error of the adjusted parameters (coordinates), was determined by taking the square roots of the diagonal elements of the variance-covariance matrix.

2.5 Literature Review

Aslan, Gundogdu and Arici (2007) identified that the shape of a parcel is amongst the significant criteria for utilizing agricultural land efficiently. The study presented the utility of symmetric parameters used in analyzing landscape fragmentation in assessing the land consolidation work. Land fragmentation is defined as the existence of a number of spatially separate plots of land which are formed as single units (Obateru, 2002). Since land parcellation is the subdivision of land ownership in a landscape while consolidation is the replacement of holdings consisting of numerous small plots (Obateru, 2002). In their study, indices which include criteria regarding shape and number of plots were used for the assessments of land consolidation studies. These indices were classified under two main headings: indices regarding size and density of parcels and indices regarding edge and shape ratio. They defined the term, 'total edge' as the total perimeter of all the parcels in the study area. They further stressed on the importance of the shape of parcels to maximize the use of agricultural land.

Ritter, Huttel, Odening and Seifert (2019) also observed that land prices are influenced by land attributes such as soil quality and distance to urban centers. Land price is also a function of the parcel size. The economies of size related to farm machinery and management, and the partially fixed transaction costs in land sales may justify price premium for larger plots. The impacts of parcel size on land prices are important because the size of sold land plots have a huge variation ranging from a few square meters is to several thousand hectares. Thus, a single estimated coefficient for the size-price relationship that may be wrong would result in over or underestimated values of plots with plot sizes far from the mean. From a normative perspective it would be useful for land vendors to know if a negative price-parcel size relationship prevails on a land market. In that case higher revenues could be generated by splitting larger plot into smaller ones. The objective of their research was to examine several hypotheses on the size-price relationship using rich data sets on land transactions in Eastern Germany. In contrast to other financial assets, land is traded on illiquid markets and sales associated with significant search and transactions costs.

Having reviewed the works from these scholars it becomes imperative that the geometry in relation to the size of any landed property should be taken into consideration during subdivision and likewise purchase of lands. However, in the above related studies, the extent of the variation in shape of test parcels the land was subjected to least squares adjustment using both methods of observation equation and condition equation with the aid of MATLAB software due to the large set of data obtained. They had a linear accuracy of 1: 115,984.609 and an angular misclosure of 8 seconds; and a perimeter of 2963.855 meters were obtained. The result obtained in their study was sufficient for any surveying and engineering studies within and around the campus. The angular variations in respect to the geometry of land was not included in their research, hence the importance of knowing the range of angular variations in land parcellation or sale becomes imperative.

3.1 Methodology

Research design also refers to the strategy that is chosen to integrate the various components of the study in a coherent and logical manner, thereby ensuring effective approach to address the research problem (Kirshenblatt-Gimblett, 2006). As such the research design brings out the conceptual structure of the research, the blueprint of data collection, measurement and analysis.

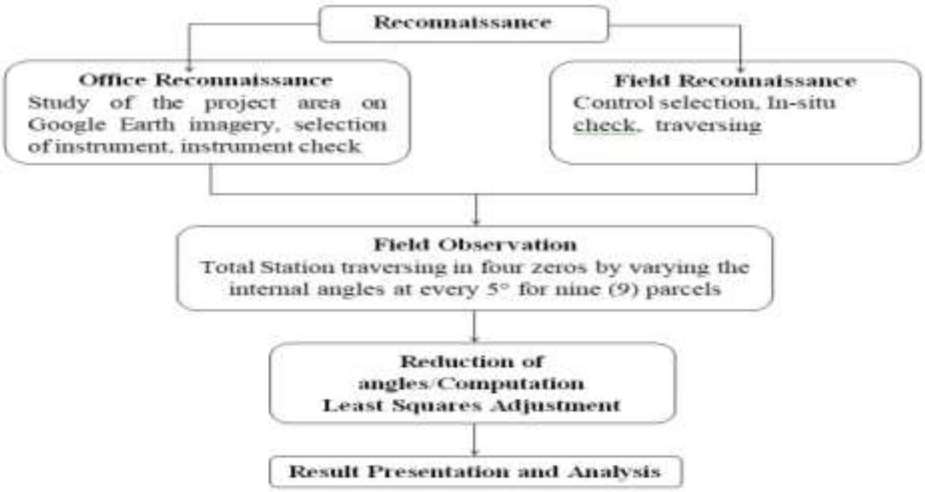

Figure 6: Shows flow chart of the study

3.2 Data search

This involves the sourcing of the existing survey information of the study area. Having obtained the aforesaid survey information, it we be necessary to identify controls closest to the study area (Arora, 2011; Ekpete, 2010).

3.3 Reconnaissance

This involves the actual site visitation for the purpose of identifying all abutments (Arora, 2011; Ekpete, 2010).

Table 1: Coordinates listing for existing control

Control Stations	Eastings (m)	Northings (m)
SVG/ GPS 002	275855.761	530903.200
WGPS 003	275992.841	530933.315
WGPS 004	275855.250	530938.002

Source: Marcus C. H, Dickson I. B. Ernest I. (2018) and Opuaji T. A. (2020)

3.4 In-situ Check

In-situ check is carried out in order to determine the positional reliability of the reference point before any survey operation is carried out. In situation (In-situ Check) test is a test carried out on the control point to be used as a reference point. The aim of any in-situ check is to ascertain the reliability and confirm the integrity of their position. The result of the in-situ check as recorded during the field operation is tabulated below:

Table 2: In-situ check on controls

Control Stations	Parameters	Computed Values	Observed Values	Difference
WGPS 004 – SVG/ GPS 002 – WGPS 003	Angle	242° 58’ 13.49”	242° 58’ 14.8”	0° 00’ 1.31”
SVG/GPS 002 – WGPS 003	Distance	42.564m	42.565m	0.001m
SVG/GPS 002 – WGPS 004	Distance	113.004m	113.001m	0.003m

3.5 Traversing

We established two points which formed our baseline and ran a traverse to coordinate those points. Thereafter, we set out other points at predefined degrees before we started running the traverse on each parcel. Four rounds of observations were made at each station.

3.6 Monumentation

Wooden pegs were used as monuments and a diagonal line was drawn on top of the pegs to indicate the center of the peg. A mark was placed at the center of the intersection of the diagonal lines. The pegs were properly placed on the ground.

3.7 Data Acquisition Method

Total Station traversing was carried out in accordance with the principle of working from a known station to the unknown stations and closing back to a known station. The Total Station was used in angular mode to acquire the horizontal circle readings of interconnected lines and the distances between them. The traverse adopted for this study was a close traverse. Four rounds of observations were made at each station.

3.8 Reduction of Observations/ Computation

3.8.1 Horizontal Angle Reduction

The horizontal angles were reduced from the horizontal circle readings using Casio Calculator. The horizontal angle at a particular station was obtained by subtracting the face left reading of back station from the face right reading of the fore station. Also, the face right of back-sight station was subtracted from face right of foresight station. If the result is negative, then 360° was added to obtain the horizontal angle. The mean of the two angles obtained was taken as the horizontal angle. The angular misclosure was checked using this formular: $(2n + 4)\ 90°$.

3.8.2 Computation of Forward Bearing

In the computation of the traverse network, the forward bearing of the traverse lines is to be computed. From the known coordinates, the initial bearing was used to compute the bearings of the other lines in the network using the formula:

Forward bearing = back bearing + observed horizontal angle.

The difference between the computed bearing derived using the observed angles and that computed from the coordinate values of the controls is the angular misclosure. The allowable angular misclosure for the study is given as $30''\sqrt{N}$.

3.8.3 Forward Computation

The adjusted bearings and true horizontal distances of the lines forming the traverse network of the perimeter have been obtained; the following computations were done to achieve results:

i. The Determination of the Initial Bearing: $\beta = \tan^{-1}\dfrac{\Delta E}{\Delta N}$ (20)

ii. Determination of the Provisional Easting and Northing

$\Delta E = L\sin\Theta$ (21)

$\Delta N = L\cos\Theta$ (22)

$N_1 = N_0 \pm \Delta N$ (23)

$E_1 = E_0 \pm \Delta E$ (24)

3.8.4 Computation of the Final Coordinates Using Least Squares

Least squares method is used in the adjustment of measured quantities containing random errors. It is assumed that all systematic errors have been removed from the observed values and that only random error and blunders which have escaped detection remain (Wolf and Ghilani, 1997; Charles, 2010). The solution of the unknown parameters or the estimate of the correction of approximate parameter vector, X, in a non-linear model is given as:

$$L^a = f(x^a) = f(x^0 + x) \tag{25}$$

$$x^a = x^0 + x \tag{26}$$

Where: x^0 = approximate value of adjusted parameters and x = correction to approximate value

$$X = -(A^T PA)^{-1} A^T PL \tag{27}$$

Where: $(A^T PA)$ = normal equation coefficient matrix

$A^T PL$ = normal equation constant

P = unit weight

$\sum_X = \sigma_0^2 (A^T PA)^{-1}$ = variance – covariance matrix of adjusted parameters (28)

Where: $\sigma_0^2 = V^T PV/n-m$ = aposteriori of unit weight, (29)

n = number of equation and

m = number of parameters.

Having obtained the variance-covariance matrix of X, the standard error of the adjusted parameters (coordinates), was determined by taking the square roots of the diagonal elements of the variance-covariance matrix. The following equations were used to generate the 'A' Matrix which was used to compute the corrections to the approximate coordinates.

$$\frac{x_i - x_j}{IJ}\partial X_i + \frac{y_i - y_j}{IJ}\partial Y_i + \frac{x_j - x_i}{IJ}\partial X_j + \frac{y_j - y_i}{IJ}\partial Y_j \tag{30}$$

$$\left[\left[\frac{y_i - y_b}{IB^2}\partial X_b + \frac{X_b - X_i}{IB^2}\partial Y_b + \frac{Y_b - Y_i}{IB^2} - \frac{Y_f - Y_i}{IF^2}\partial X_i + - \frac{X_i - X_b}{IB^2} - \frac{X_i - X_f}{IF^2}\partial Y_i + \frac{Y_f - Y_i}{IF^2}\partial X_f + \frac{X_i - X_f}{IF^2}\partial Y_f\right]\right] \tag{31}$$

The following matrices were generated after using the above equations:

$$A = \begin{bmatrix} 0.948 & -0.319 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.348 & 0.937 & -0.348 & -0.937 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.984 & -0.18 & -0.984 & 0.18 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.348 & -0.937 & 0.348 & 0.937 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.888 & -0.459 \\ -9341.937 & -3147.115 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9487.255 & 6988.333 & -6340.14 & 2353.604 & 0 & 0 & 0 & 0 \\ -6340.14 & 2353.604 & 5671.429 & -6791.277 & 812.402 & 4437.673 & 0 & 0 \\ 0 & 0 & 812.402 & 4437.673 & -7151.905 & -2083.336 & 6339.503 & -2354.337 \\ 0 & 0 & 0 & 0 & 6339.503 & -2354.337 & -7863.642 & 5301.119 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1524.139 & -2946.782 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 \\ 0 \\ 0.001 \\ 0.001 \\ 0.001 \\ 0.004027 \\ 0.007361 \\ -0.007361 \\ 0.000166 \\ -0.000833 \\ -0.003888 \end{bmatrix}$$

Corrections to the Approximate Coordinates:

$$X = \begin{bmatrix} 0.00000183339 \\ -0.00000415676 \\ 0.00026183781 \\ 0.00070717139 \\ 0.00081845281 \\ 0.0006042458 \\ 0.00073375883 \\ 0.00037819636 \end{bmatrix}$$

4.1 Result Presentation and Analysis

The area of each parcel was computed and the results for the 18 varying parcels are displayed on the table below in Universal Transverse Mercator (UTM) zone 32N projection system.

Table 3: Table Showing Results from Parcel 1 (100 x 100m)

Parcel ID	Back Comp. Coordinates		Back Comp. Area (m^2)	Adjusted Coordinates		Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
	N (m)	E (m)		N (m)	E (m)			
BASE 1	530931.730	275875.422		530931.729	275875.425		±0.000, ±0.001	
1A	531030.734	275890.214		531030.730	275890.217		±0.029, ±0.079	
1B	531015.940	275989.222		531015.937	275989.221		±0.074, ±0.076	
BASE 2	530916.937	275974.428	10,020.879	530916.936	275974.427	10,020.229	±0.008, ±0.004	0.650

Table 4: Table Showing Results from Parcel 2 (100 x 100m)

Parcel ID	Back Comp. Coordinates		Back Comp. Area (m^2)	Adjusted Coordinates		Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
	N (m)	E (m)		N (m)	E (m)			
BASE 1	530931.726	275875.420		530931.729	275875.425		±0.000, ±0.001	
2A	531031.641	275881.516		531030.730	275890.217		±0.000, ±0.001	
2B	531016.864	275980.521		531015.937	275989.221		±0.000, ±0.001	
BASE 2	530916.948	275974.404	9,981.324	530916.936	275974.427	9,982.705	±0.008, ±0.004	-1.897

Table 5: Table Showing Results from Parcel 3 (100 x 100m)

Parcel ID	Back Comp. Coordinates		Back Comp. Area (m^2)	Adjusted Coordinates		Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
	N (m)	E (m)		N (m)	E (m)			
BASE 1	530931.730	275875.422		530931.729	275875.425		±0.000, ±0.001	
3A	531031.792	275872.799		531031.809	275872.800		±0.000, ±0.001	
3B	531017.000	275971.800		531017.017	275971.801		±0.000, ±0.001	
BASE 2	530916.940	275974.424	9,867.385	530916.936	275974.427	9,869.282	±0.008, ±0.004	-1.897

Table 6: Table Showing Results from Parcel 4 (100 x 100m)

Parcel ID	Back Comp. Coordinates		Back Comp. Area (m^2)	Adjusted Coordinates		Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
	N (m)	E (m)		N (m)	E (m)			
BASE 1	530931.730	275875.420		530931.729	275875.425		±0.000, ±0.001	
4A	531031.189	275864.086		531031.190	275864.086		±0.000, ±0.001	
4B	531016.399	275963.084		531016.400	275963.084		±0.000, ±0.001	
BASE 2	530916.949	275974.420	9,678.247	530916.936	275974.427	9,679.037	±0.008, ±0.004	-0.790

Table 7: Table Showing Results from Parcel 5 (100 x 100m)

Parcel ID	Back Comp. Coordinates		Back Comp. Area (m^2)	Adjusted Coordinates		Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
	N (m)	E (m)		N (m)	E (m)			
BASE 1	530931.732	275875.419		530931.729	275875.425		±0.000, ±0.001	
5A	531029.816	275855.460		531029.817	275855.460		±0.000, ±0.001	

5B	531015.024	275954.464		531015.025	275954.464		+0.000, +0.001	
BASE 2	530916.940	275974.423	9,415.475	530916.936	275974.427	9,415.738	+0.008, +0.004	0.263

Table 8: Table Showing Results from Parcel 6 (100 x 100m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530931.732	275875.419		530931.729	275875.425		+0.000, +0.001	
6A	531027.709	275846.987		531027.709	275846.987		+0.000, +0.001	
6B	531012.916	275945.988		531012.916	275945.988		+0.000, +0.001	
BASE 2	530916.932	275974.421	9,081.512	530916.936	275974.427	9,081.371	+0.008, +0.004	0.141

Table 9: Table Showing Results from Parcel 7 (100 x 100m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530931.731	275875.419		530931.729	275875.425		+0.000, +0.001	
7A	531024.868	275838.730		531027.709	275846.987		+0.000, +0.001	
7B	531010.074	275937.734		531012.916	275945.988		+0.000, +0.001	
BASE 2	530916.935	275974.425	8,678.245	530916.936	275974.427	8,678.257	+0.008, +0.004	-0.012

Table 10: Table Showing Results from Parcel 8 (100 x 100m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530931.731	275875.419		530931.729	275875.425		+0.000, +0.001	
8A	531021.312	275830.752		531027.709	275846.987		+0.000, +0.001	
8B	531006.520	275929.752		531012.916	275945.988		+0.000, +0.001	
BASE 2	530916.939	275974.419	8,207.805	530916.936	275974.427	8,208.382	+0.008, +0.004	-0.577

Table 11: Table Showing Results from Parcel 9 (100 x 100m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530931.731	275875.418		530931.729	275875.425		+0.000, +0.001	
9A	531017.082	275823.111		531017.084	275823.110		+0.000, +0.001	
9B	531002.288	275922.117		531002.290	275922.116		+0.000, +0.001	
BASE 2	530916.936	275974.423	7,676.419	530916.936	275974.427	7,676.545	+0.008, +0.004	-0.126

Table 12: Table Showing Results from Parcel 1 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.520	275982.590		530896.520	275982.589		+0.001, +0.000	
1A	530866.519	275977.098		530866.519	275977.097		+0.000, +0.001	
1B	530874.752	275932.126		530874.752	275932.125		+0.000, +0.001	
BASE 2	530904.752	275937.618	1,394.421	530904.752	275937.617	1,394.395	+0.001, +0.000	0.026

Table 13: Table Showing Results from Parcel 2 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.520	275982.590		530896.520	275982.589		+0.001, +0.000	
2A	530867.112	275974.504		530867.113	275974.504		+0.000, +0.001	
2B	530875.344	275929.533		530875.345	275929.533		+0.000, +0.001	
BASE 2	530904.752	275937.618	1,389.082	530904.752	275937.617	1,389.029	+0.001, +0.000	0.053

Table 14: Table Showing Results from Parcel 3 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.521	275982.588		530896.520	275982.589		+0.001, +0.000	
3A	530867.928	275971.976		530867.930	275971.977		+0.000, +0.001	
3B	530876.161	275927.004		530876.163	275927.005		+0.000, +0.001	
BASE 2	530904.752	275937.622	1,373.156	530904.752	275937.617	1,373.090	+0.001, +0.000	0.066

Table 15: Table Showing Results from Parcel 4 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)

BASE 1	530896.521	275982.588		530896.520	275982.589		+0.001, +0.000	
4A	530868.967	275969.520		530870.214	275969.521		+0.000, +0.001	
4B	530877.200	275924.544		530877.202	275924.545		+0.000, +0.001	
BASE 2	530904.755	275937.613	1,346.877	530904.752	275937.617	1,346.655	+0.001, +0.000	0.222

Table 16: Table Showing Results from Parcel 5 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.521	275982.589		530896.520	275982.589		+0.001, +0.000	
5A	530870.212	275967.170		530870.214	275962.171		+0.001, +0.001	
5B	530878.445	275922.191		530878.447	275922.192		+0.001, +0.001	
BASE 2	530904.758	275937.612	1,310.399	530904.752	275937.617	1,310.061	+0.001, +0.000	0.338

Table 17: Table Showing Results from Parcel 6 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.535	275982.596		530896.520	275982.589		+0.001, +0.000	
6A	530871.670	275964.948		530871.670	275964.948		+0.001, +0.001	
6B	530879.861	275919.960		530881.502	275919.960		+0.000, +0.000	
BASE 2	530904.740	275937.602	1,263.670	530904.752	275937.617	1,206.838	+0.001, +0.000	0.070

Table 18: Showing Results from Parcel 7 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.535	275982.596		530896.520	275982.589		+0.001, +0.000	
7A	530873.316	275962.834		530873.320	275962.838		+0.002, +0.002	
7B	530881.498	275917.845		530881.502	275917.849		+0.002, +0.002	
BASE 2	530904.723	275937.613	1,206.441	530904.752	275937.617	1,206.838	+0.001, +0.000	0.397

Table 19: Showing Results from Parcel 8 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.534	275982.596		530896.520	275982.589		+0.001, +0.000	
8A	530875.096	275960.907		530875.099	275960.910		+0.002, +0.002	
8B	530883.312	275915.926		530883.315	275915.929		+0.002, +0.002	
BASE 2	530904.737	275937.629	1,141.974	530904.752	275937.617	1,142.127	+0.001, +0.000	0.153

Table 20: Showing Results from Parcel 9 (30 x 45m)

Parcel ID	Back Comp. Coordinates N (m)	E (m)	Back Comp. Area (m^2)	Adjusted Coordinates N (m)	E (m)	Adj. Comp. Area (m^2)	Standard Error ΔE(m), ΔN(m)	Difference in Area (m^2)
BASE 1	530896.533	275982.597		530896.520	275982.589		+0.001, +0.000	
9A	530877.031	275959.147		530877.034	275959.150		+0.002, +0.002	
9B	530885.276	275914.173		530885.279	275914.176		+0.002, +0.002	
BASE 2	530904.743	275937.647	1,069.096	530904.752	275937.617	1,069.162	+0.001, +0.000	0.066

Table 21: Table showing the Parcels, varying misclosures and geometry (100 x 100m)

Parcel	Angle Variation	Angular Misclosure	Perimeter (m)	Linear Accuracy	Area (m^2)	Difference in Area (m^2)	Number of 50 x 100 plot	Number of 50 x 100 plot lost
1	90°	-1”	400.40	1: 227,878	10,020.010	0.000	21.571	0.000
2	85°	58.38”	400.40	1:108,733	9,981.881	38.129	21.489	0.082
3	80°	00”	400.40	1:45,938	9,867.784	152.226	21.243	0.328
4	75°	-1.5”	400.40	1:41,728	9,678.586	341.424	20.836	0.735
5	70°	-4”	400.40	1:229,579	9,415.729	604.281	20.270	1.301
6	65°	-6”	400.40	1:9,111	9,081.213	938.797	19.550	2.021
7	60°	-3”	400.40	1:114,823	8,677.583	1,342.427	18.681	2.890
8	55°	32.7”	400.40	1:448,190	8,207.912	1,812.098	17.670	3.901
9	50°	-6”	400.40	1:229,591	7,675.773	2,344.237	16.524	5.047

Table 22: Table showing the Parcels, varying misclosures and geometry (30 x 45m)

Parcel	Angle Variation	Angular Misclosure	Perimeter (m)	Linear Accuracy	Area (m^2)	Difference in Area (m^2)	Number of 50 x 100 plot	Number of 50 x 100 plot lost
1	90°	0.3”	152.44		1,394.421	0.000	3.00	0.000

2	85°	2”	152.44	1:232,148	1,389.082	5.339	2.99	0.011
3	80°	-2”	152.44	1:232,164	1,373.156	21.185	2.96	0.046
4	75°	-1”	152.44	1:77,434	1,346.885	47.515	2.90	0.102
5	70°	2.5”	152.44	1:114,754	1,310.404	84.096	2.82	0.181
6	65°	39.6”	152.44	1:46,025	1,263.670	130.649	2.72	0.281
7	60°	39”	152.44	1:17,392	1,206.441	186.820	2.60	0.402
8	55°	16”	152.44	1:8,006	1,141.996	252.182	2.46	0.543
9	50°	9”	152.44	1:13,618	1,069.096	326.238	2.30	0.702

Tables 3 to 11 shows the results for the 100×100m parcels while tables 12 to 20 shows the results for 30x45m parcels comparing the products of the unadjusted coordinates of each parcel and that of the least square adjustment coordinates with their standard errors and difference in areas. Tables 21 and 22 shows that the angular variation for the varying geometry is 5° ranging from 90° to 50° with a total of 9 parcels each for the 100x100m and 30x45m respectively. Hence, the misclosure in every angle variation to both the tables 21 and 22 changed. Although one of the angular variations in the table 21 without misclosure having 00". As the angle variation decreases in degree's so the area and the number of plot decreases. As such the decrease of both number of plots and areas is the increase of number of plots lost.

Table 21 have angular misclosures ranging from -6’’ to 58.38’’ with a constant perimeter of 400.40m, linear accuracy ranging from 1:9,111 to 1:448,190, area ranging from 10,020.010m^2 (1.002ha) to 7,675.773m^2 with a difference in area ranging from 38.129m^2 to 2,344.237m^2 that is an equivalent of 0.082 to 5.047 plots lost using 464.515m^2 an equivalent of 15.24m (50ft) by 30.48m (100ft) as a standard plot in South East and South South states of Nigeria. Similarly, table 22 have angular misclosures ranging from -2’’ to 39.6’’ with a constant perimeter of 152.44m, linear accuracy ranging from 1:8,006 to 1:232,164, area ranging from 1,394.421m^2 to 1,069.096m^2 with a difference in area ranging from 5.339m^2 to 326.238m^2 that is an equivalent of 0.011 to 0.702 plots lost. Lastly, the angle variation determines the plot size and the area of a parcel but with uniform perimeter. Even when it is plotted the variation in size and shape will eventually be visible as shown figure 7.

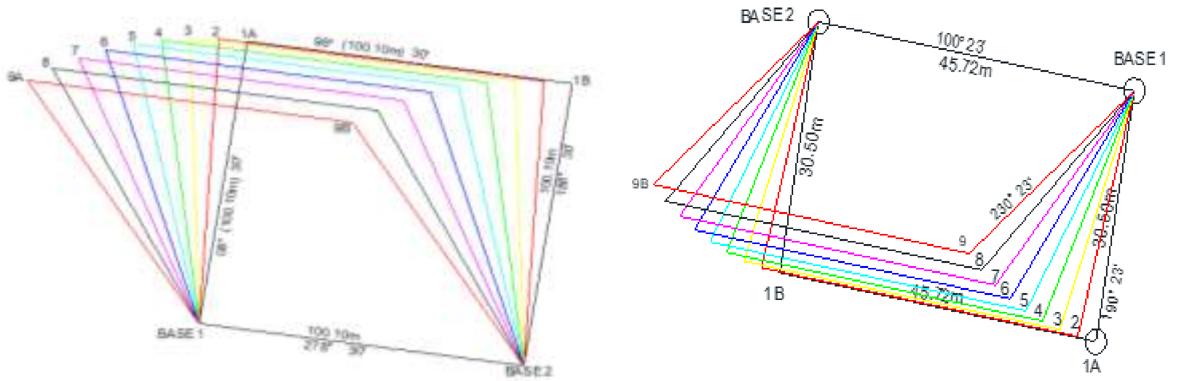

Figure 7: Plans of the different parcels superimposed on each other (100x100m) and (30x45m)

Considering the shapes and sizes of the parcels in tables 21 and 22 differs in terms of pattern and trend. From the results display table 21 shows that the parcel is fairly large while table 22 is a smaller area.

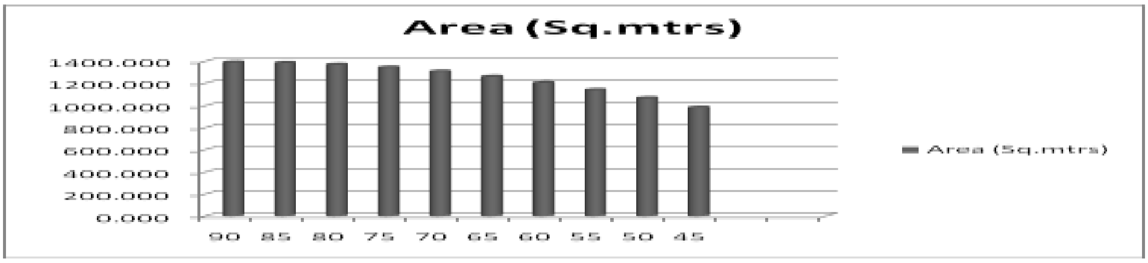

Figure 8: Statistical Model of Variations in Area

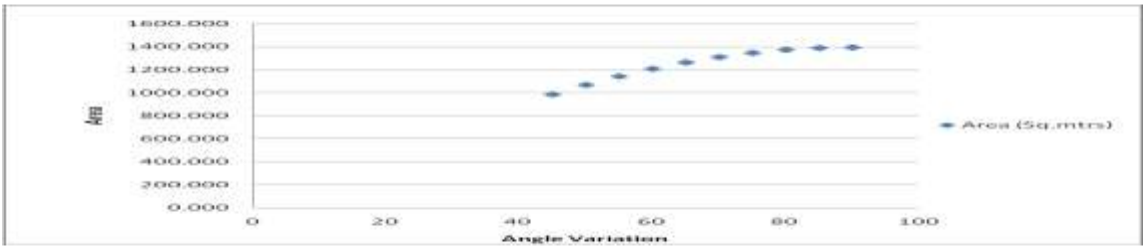

Figure 9: Graphical Representation of variations in Area

5.1 Summary

Traversing was carried out at varying geometry of land with the aim of determining the area of each parcel of land. The survey was done on a 30×45m plot of land and on a 100×100m parcel of land. On each parcel, the perimeter was kept constant; this was made possible by the establishment of a baseline. A total of 18 parcel of land were surveyed of varying geometry, nine (9) for the smaller plot of land and nine (9) for the bigger plot. Different angular misclosures and linear accuracy for each parcel was calculated and tabulated in tables 21 and 22 of this study.

The area of each parcel has also been computed and tabulated. This research will benefit both the researcher and the society, and will show that to every variation in the internal angle of a landed property, there's a corresponding change in the area of that landed property. This research will educate both the academics and land professionals on the importance of the geometry on any proposed layout design. From the results of this study, we can say that 70^{o} could be said to be the acceptable angular deviation without having much effect on the area.

5.2 Conclusion

It is important to restart the overall objectives of this study are as follows; firstly, to carry out traverse on different geometry of a land with the same perimeter and produce boundary coordinates. Secondly, to determine the adjusted coordinates using least squares and the area of each parcel. Thirdly, to produce the plans of the different parcels superimposed on each other and to present both tabular and graphical form, the change in area as a result of the variations in angles.

To achieve the objectives, relevant research works were reviewed that shows that varying geometry area determination is important to land professionals. This study has added more value to researchers by enlightening the land professionals on the necessity of taking into consideration, the geometry of any parcel of land in other to maximize the use of such landed property. From the results of this study, it is obvious that the more inclined the shape of a land is, the lesser the value of such land.

There was a drastic drop in the area of land when the internal angle reduced from 90° to 50°. For smaller parcels of 30×45, 2.9 plots of land were lost when the internal angle reduced from 90° to 50° which is indeed a great loss for the owner of the land. From the larger parcel of 100×100, 2.9 plots of land were lost when the internal angle reduced from 90° to 50° which is indeed a great loss for the owner of the land. If a plot of land is sold at ₦5,000,000 it implies that ₦15,000,000 would be lost because of the non-adherence to the principles of geometry. The effect of the reduction in the interior angle of land is more on larger parcels than smaller parcels land.

5.3 Recommendation

The research has revealed salient points regarding the effect of varying the geometry of lands in relation to area determination in Nigeria. The comparison of smaller and a fairly large parcels should not be overlooked as it solves the problems of doubt from the result of computed parcels. The results of the work should be readily accessed by built professionals as a contribution into best practices for countries that sales or leases of properties in plots rather than square meters.

If implemented, this work will benefit owners of properties value for money in the aforesaid countries and saves the cost of multiple surveys in a bid to achieve a different result since perimeter was more considered rather than the area. It will benefit the professionals by helping to facilitate swift survey for effective land registration and management, easier and faster than it is possible in the existing practice. The Government will also benefit by the possible drastic reduction of land dispute and tenure related challenges in terms of paying so much for lesser coverage of land.

It should be recommended that the surveyors and other land professionals adopt a maximum angular deviation during subdivision, lease and sale of landed property. If possible, a law should be passed through the appropriate professional bodies on the maximum angular deviation on properties of smaller dimensions.

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