A New Method to Derive Normal Height from GPS Height Based on Neural Network

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Key words: neural network, algorithm of BP (back propagation), GPS height, CFM&NNM.

ABSTRACT

The adjusted GPS height is the height above the surface of the WGS-84ellipsoid. In China, however, normal height, which is the height above geoid calculated using the mean normal gravity along the plumb line, is used in engineering applications. Thus, it is necessary to convert a GPS height into a normal height. Normally, the conicoid fitting method (CFM) and the neural network method (NNM) are used for this purpose. But, each of them has its own advantages and disadvantages. After studying these two methods, a new method (abbr. CFM&NNM) is conceived that combines the advantages of both the conicoid fitting (CFM) and neural network method (NNM). This paper discusses the structure of the BP neural network and detailed algorithm of the CFM, NNM and CFM&NNM method.

A practical engineering example is used to study the three different methods. In a city's D-order GPS network (about 300km^2) , 44 GPS points have third-order elevations obtained by geodetic leveling survey. Removing the 4 points which were found to contain gross error, the new method is tested with the rest 40 points. We take 10 evenly scattered points as a study-group to train the neural network and the other 30 points as a work-group to check the effectiveness of the trained neural network. Comparison of the three methods is discussed. After studying with the study-group , the working mean square error of the work-group is about ±7.8mm by CFM, about ±6.9mm by NNM, and about ± 5.5mm by CFM&NNM. It is demonstrated that the combined CFM&NNM method produces better results than either the CFM or the NNM in deriving normal height from GPS height.

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NOMENCLATURE

BP	An algorithm of a back propagation neural network
CFM	Conicoid fitting method
CFM&NNM	Conicoid fitting and neural network method
GPS	Global positioning system
H_{GPS}	GPS height
H_{Nor}	Normal height
NNM	Neural network method
$W_{ji}(t)$	Weight between unit i and unit j when cycle time is t
	Height abnormity
$\Delta\xi$	Error of the height abnormity

1. INTRODUCTION

GPS (Global Positioning System) surveying technique has been widely used in precise engineering surveys. It is well-known that GPS provides more accurate horizontal positions than vertical position. Much research has been done to improve the accuracy of GPS-derived elevations.

The adjusted GPS height (H_{GPS}) is the height above the WGS-84 ellipsoid. In China, however, the normal height (H_{Nor}.), which is the height above geoid calculated using the mean normal gravity along the plumb line, is used in engineering applications. It is, therefore, necessary to convert H_{GPS} into H_{Nor}. The difference between them is called height abnormity ξ:

$$\xi = H_{HPS} - H_{Nor} \qquad (1)$$

If accurate H_{Nor} can be achieved by adjusting and converting H_{GPS}, it may be used to replace the laborious geodetic leveling work. The CFM and NNM are often used for converting H_{GPS}.

1.1 Conicoid Fitting Method (CFM)

Its main idea is to design a set of survey marks where both the normal height H_{Nor} and GPS height H_{GPS} are known, and then the height abnormity ξ is modeled by a polynomial of second degree, as follows:

$$\xi(x, y) = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2 \qquad (2)$$

x, y a mark's horizontal coordinates

$a_0, a_1......a_5$ unknown coefficients

So more than six marks with known H_{GPS} and H_{Nor} are needed. Based on the adjustment results of the CFM, we can adjust the weight of observations (the known points) so that the weight of an observation with gross error is small, even zero, to enhance the accuracy of the adjustment. The anti-error conicoid fitting method may produce better results.

1.2 Neural Network Method (NNM)

Artificial neural network is a relatively new branch of science. It is a highly simplified model of a complicated bio-neural system. Since the 1980's, scientists in many fields (including engineering) have spent tremendous effort in studying artificial neural network and made remarkable accomplishments. The NNM to convert H_{GPS} is a self-adapted mapping method with no hypothesis, which can greatly reduce model error. Its accuracy is better than that of CFM.

The advantages and disadvantages of both the CFM and the NNM are listed in Table 1 below:

Table 1: The comparison between CFM and NNM

Method	Advantage	Disadvantage
CFM	1.Calculation simple and fast 2.If a marks' value has gross error, the anti-error CFM can reduce the influence of the gross error after adjustment.	1.With model error 2.Fitting accuracy is not high enough
NNM	1. No model error 2. Fitting accuracy is high	1.Calculation complicated 2.Initial weight selection influences the convergence speed and result

After comparing these two methods, a new method, called CFM&NNM, to convert H_{GPS} is conceived. In this paper we will discuss the structure and algorithm of the BP neural network and the idea of CFM&NNM to convert H_{GPS}. An engineering example is given to demonstrate the three different methods.

2. THE STRUCTURE AND ALGORITHM OF THE BP NEURAL NETWORK

Recently there are more than 40 types of neural network models. For Converting H_{GPS}, we adopt a multi-layer fore-feedback BP structure. BP (back propagation) algorithm is widely used in engineering, which is advisor-trained and adopts the training pattern of minimum square error.

2.1 The Structure of the BP Neural Network

The structure of BP neural network is shown in chart 1. It can be divided into five layers: the input transformation layer, the input layer, the hidden layer, the output layer and the output transformation layer. For an ordinary engineering application, the input transformation layer and output transformation layer are needed because the input and output of Sigmoid standard active function f(x) ranges from 0 to 1.

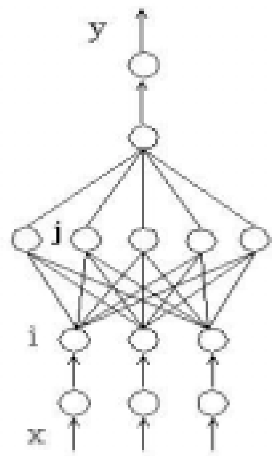

Chart 1: A single hidden layer model of a BP network

2.2 The Algorithm of the BP Neural Network

The computation formula for the input transformation layer and the output transformation layer is different depending on the specific engineering application. It can be coded using a computer language to automate the computation. For every unit of the input layer, its input and output are the same. While for every unit of the hidden layer or the output layer, for example, unit j in Chart 1, its input value I_{Pj} and output value O_{Pj} can be described as

$$I_{pj} = \sum_{i=1}^{n} W_{ji} O_{pi} \qquad (3)$$

$$O_{pj} = f(I_{pj}) \qquad (4)$$

$$f(x) = \frac{1}{1+e^{-x}} \qquad (5)$$

p Serial number of an example to train the BP network
i Unit number down to unit j layer
W_{ji} Connection weight between unit i and unit j
f(x) The active function of unit j

We define formula 6 below as an objective function

$$E = \frac{1}{2}\sum_{i=1}^{p}(y_k - y_k')^2 \qquad (6)$$

y_k the unit's expected output

y_k' the unit's true output

Now the problem becomes to search for the extreme value of equation (6) with no constraints, i.e.:

$$E(W) = \min \qquad (7)$$

Adopt the quickest descending method and change the weight with the objective function's descending to reach the minimum value point. To increase the calculation speed, the training speed self-adapted method is used. The formula is

$$W_{ji}(t+1) = W_{ji}(t) - \eta \frac{\partial E}{\partial w} + \alpha (W_{ji}(t) - W_{ji}(t-1)) \qquad (8)$$

η training speed
α momentum coefficient

2.3 The Calculation Process of BP Algorithm

1) Initialize weights $W_{ji}(0)$ with smaller non-zero random value;

2) Use a number of examples to train the network, repeat the following process until the required precision is achieved;
(1) Forward process: To calculate with given value from the input layer to the hidden layer and output layer, until the output is achieved;
(2) Backward process: To calculate the error between true outputs and expected outputs, and based on the errors, adjust weights from the output layer backward layer by layer.

3 THE IDEA OF CFM&NNM

From Table 1, since CFM and NNM both have their own advantages and disadvantages, the author conceives a new method, which combines their advantages, to convert H_{GPS}. The procedure is as follows:

1) Assume there are n points, of which n_1 points' values of H_{GPS} and H_{Nor} are known, and n_2 (n_2=n-n_1) points' value of H_{Nor} need to be calculated.
2) Based on the n_1 points' H_{GPS} and H_{Nor}, model all points' height abnormity (ξ) by CFM.
3) Calculate n_1 points' error of height abnormity as follows
$$\Delta\xi = \xi_0 - \xi$$
where $\xi_0 = H_{GPS} - H_{Nor}$
4) Use the above n_1 points' information $(x_i, y_i, \xi_i, \Delta\xi_i \quad i = 1,2,3......n_1)$ as a training sample. The BP network is trained by this sample.
5) The n_2 points' error of height abnormity ($\Delta\xi$) can be calculated by the trained BP network. The normal height can be calculated by
$$H_{Nor} = H_{GPS} - \xi_0 = H_{GPS} - (\xi + \Delta\xi)$$
where ξ is calculated by CFM and $\Delta\xi$ by NNM.

4 AN EXAMPLE

A city's D-order GPS network (about 300km^2) has 96 observation points, of which, 44 GPS points have third-order elevations obtained by geodetic leveling survey. After adjustment, the mean square error of one kilometer in level survey is $\pm 2.4mm$ (the limit is $\pm 6.0mm$). Among the 44 points, there are 4 points which were found to contain gross error, and they are points No.13 No.27 No.46 and No.77. Removing the 4 points, the new method is tested with the rest 40 points.

Chart 2: Distribution of benchmarks in the D-order GPS network

In Chart 2, we take 10 evenly distributed points as a group to train the neural network and the other 30 points as another group to check the effectiveness of the trained neural network. Comparison of the three methods is discussed below.

4.1 The CFM Method

The unknown coefficients $a_0, a_1, ..., a_5$ in formula (2) can be obtained by CFM, and then computing all 44 points' height abnormity (ξ) by formula (2). The result is shown in Table 2 below.

Table 2: The Results of CFM

No.	$\Delta\xi = \xi_0 - \xi$ (mm)	No.	$\Delta\xi = \xi_0 - \xi$ (mm)	No.	$\Delta\xi = \xi_0 - \xi$ (mm)	No.	$\Delta\xi = \xi_0 - \xi$ (mm)
1	-1.7	14	-4.0	20	-5.7	6	-6.0
18	0.0	80	3.6	26	0.1	10	-5.3
82	3.5	31	-5.3	30	-5.7	15	-7.3
63	0.9	24	5.2	32	1.1	16	-8.5
9	0.6	48	-12.8	34	-0.7	17	-6.3
51	-3.5	69	-12.2	39	-9.3	62	-11.4
45	5.1	11	2.9	50	2.8	64	-6.9
37	-1.3	33	-2.1	52	-6.3	65	7.4
22	3.2	61	-10.0	54	-15.1	66	-3.3
59	-6.9	5	12.6	56	-10.8	210	11.5
Note	Training mean square error $n_1=10$ $m_1= \pm 3.4$mm	Note	Working mean square error $n_2=30$ $m_2= \pm 7.8$mm				

No.	point number
$\Delta\xi$	Difference of height abnormity

4.2 The NNM Method

After we use the NNM to convert H_{GPS} for over one thousand times, the ideal BP neural network structure is the one that has two units (x,y) in the input layer, 15 units in the hidden layer and one unit (ξ) in the output layer. In a training process, the training mean square error acts as the convergence standard. The results can be seen in Table 3 below.

Table 3: The results of NNM

Training mean square error	±10.0mm	±5.0mm	±4.0mm	±2.0mm	±1.0mm
Cycle times	1874	3256	3727	7001	11042
Working mean square error	±9.6mm	±8.4mm	±8.1mm	±7.3mm	±6.9mm

Table 3 shows that when the training mean square error is ±1.0mm, the neural network precision can reach ±6.9mm, which is higher than that of CFM. The biggest shortcoming of the NNM is that the result is not so stable and the final results are largely influenced by the initial weight adopted.

4.3 The CFM&NNM Method

We take the training mean square error as the convergence standard with the example. $W_{ji}(0)$ is given non-zero random value. The value of $W_{ji}(0)$ has an important effect on convergence speed and the final result, so we have trained the neural network for more than one hundred times. The difference of every two times' results (such as the cycle times the result of height abnormity by CFM&NNM working mean square error, etc) is very small, and in all cases, the computation converged. The results of the process are listed in Table 4 below.

Table 4: The results of CFM&NNM

Training mean square error(mm)		±2.8	±2.4	±2.0	±1.6	±1.2	±0.8	±0.4
NO.1	Cycle times	491	703	4978	6015	6700	7632	8821
	Working mean square error (mm)	±7.6	±6.4	±5.5	±5.8	±5.9	±6.1	±6.2
NO.2	Cycle times	505	631	4645	5457	6053	6804	8998
	Working mean square error (mm)	±7.7	±6.5	±5.4	±5.6	±5.7	±6.0	±6.1
NO.3	Cycle times	570	688	5324	6670	7159	8383	9370
	Working mean square error (mm)	±7.7	±6.4	±5.5	±5.7	±5.8	±6.0	±6.2

From Table 4, we can derive three obvious conclusions:

1) The cycle times and working mean square error in the three different cases is very close, so we can say that the BP structure discussed in this paper is very stable;
2) When the training mean square error is ± 2.0mm, the working mean square error reaches the minimum value of ± 5.5mm;
3) When the training mean square error is less than ± 2.0mm, the smaller the training mean square error, the bigger the working mean square error.

4.4 Theoretical Analysis of CFM&NNM

The third conclusion above is that when the training mean square error is less than ± 2.0mm, the smaller the training mean square error, the bigger the working mean square error. Let us analyze this phenomenon in more detail below.

In fact, the CFM&NNM method constructed in this paper can be used for detecting the model error of the CFM with the help of the neural network. This can be explained by the BP network structure of the CFM&NNM: among the parameters of the input layer of the BP network, there is one parameter ξ, which is the result of height abnormity by CFM, and the parameter of the output layer is the difference of height abnormity $\Delta\xi = \xi_0 - \xi$ which is the difference between the height abnormity ξ by CFM and its true value ξ_0. Some conclusions may be drawn based on Table 2 and Table 4. In CFM, we can achieve the results with the training mean square error of ± 3.4mm, of which, about 40 percent, i.e., about ± 1.4mm, is model error, and the remaining (about ± 2.0mm) is observation error. So in CFM&NNM, when the neural network's training error is ± 2.0mm, the model error is removed. Here the working mean square error reaches minimum, and the average value is ± 5.5mm, which accounts to 70 percent of the working mean square error (± 7.8mm) by CFM. The method is thus very effective. Now if we continue to reduce the training mean square error by taking the observation error as the model error, the results will be worse. But overall, they are still better than those of CFM and NNM. In practical applications, when it is difficult to determine the relation between model error and observation error, experiences should be consulted.

In an area of 300 km^2, only 10 GPS points with H_{Nor} are needed and the precision can reach ± 5.5mm, if CFM&NNM is used. When the training mean square error is ±2.0mm, the results by CFM&NNM can be seen in Table 5. The formula of the difference of height abnormity in Table 5 is as follows:

$$\Delta\xi = \xi_0 - \xi_0' \qquad 9$$

ξ_0 the known height abnormity

ξ_0' the height abnormity calculated by CFM&NNM

Table 5: The results by CFM&NNM

Point number	Difference of height abnormity $\Delta\xi$ (mm)	Point number	Difference of height abnormity $\Delta\xi$ (mm)	Point number	Difference of height abnormity $\Delta\xi$ (mm)
14	-4.3	20	-4.9	6	-6.4
80	1.2	26	-4.7	10	-8.2
31	-4.4	30	-6.2	15	-7.7
24	0.9	32	-0.6	16	-8.5
48	-9.6	34	-4.2	17	-5.8
69	-2.0	39	0.7	62	-1.8
11	0.7	50	0.2	64	-6.0
33	-5.1	52	-2.5	65	8.6
61	-0.6	54	-4.8	66	-1.1
5	11.5	56	-0.6	210	7.7
Note	Working mean square error n_2=30 m_2= ±5.5mm				

Comparing Table 5 and Table 2, it is obvious that the CFM&NNM method produces better results than the CFM method. For example, the total number of points, whose difference of height abnormity is larger than 10 mm, is 8 by CFM, and is only 1 by CFM&NNM.

5 CONCLUSION

Compared with CFM and NNM, CFM&NNM produces more accurate results in converting GPS height H_{GPS} into normal height H_{Nor}. More work should be done to evaluate its effectiveness for larger engineering projects with more complicated topography. The disadvantage of the method based on neural network is that the result is not so stable and the final results are largely influenced by the initial weight $W_{ji}(0)$. But the BP structure of the CFM&NNM discussed in this paper is stable in calculating $\Delta\xi$ (error of the height abnormity) with the selection of the initial weight $W_{ji}(0)$ having no effect on the results. Thus it is recommended for use in engineering projects.

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